# The Language of Mathematics in Science 

## Teaching Approaches


$n$ The Association
$n$ for Science Education

# THE LANGUAGE OF MATHEMATICS IN SCIENCE 

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## Introduction

This publication provides teachers, and others with an interest in science education, with authentic accounts of classroom practice describing the teaching of mathematics in science lessons to pupils aged 11-16. It features eight different accounts, whiich are summarised in Section A and presented in full in Section B. The accounts illustrate ways in which different teachers have addressed problems encountered by students in using mathematical ideas in science, and provides insight into student responses to their approaches. The commentaries in Section A provide an overview of each account and identify key issues. They are designed to stimulate reflection and discussion, and to help the reader select specific detailed teacher accounts from Section B for further reading.

## Background

This publication was produced as part of the ASE project The Language of Mathematics in Science, which was funded by the Nuffield Foundation and aims to support teachers of $11-16$ science in the use of mathematical ideas in the science curriculum. This is an area that has been a matter of interest and debate for many years. Some of the concerns have been about inconsistency in terminology and approaches between the two subjects. The project built on the approach of a previous ASE project, The Language of Measurement (also funded by the Nuffield Foundation), and the purpose is similar: to achieve a greater clarity and agreement about the way the ideas and terminology are used in teaching and assessment.

Two publications have been produced during the project. This publication, The Language of Mathematics in Science: Teaching Approaches, uses teachers' accounts to illustrate examples of teaching approaches with examples of how children respond to different learning activities, and it also outlines various ways that science and mathematics departments are working together. This publication complements The Language of Mathematics in Science: A Guide for Teachers of 11-16 Science, which provides an overview of relevant ideas in secondary school mathematics and where they are used in science. It aims to clarify terminology, and to indicate where there may be barriers to student understanding. It also includes explanations of key ideas and terminology in mathematics, along with a glossary of terms.

## Purposes

The accounts describe a range of different student age groups in a range of different types of school. They provide:

- examples of collaboration between mathematics and science teachers
- science learning activities designed to address specific mathematical ideas
- examples of responses of individual students and groups to lesson activities.

The purpose of presenting these descriptions of classroom practice alongside the guidance materials is to:

- illustrate different ways of teaching a particular aspect of mathematics
- promote individual reflection on the use of mathematics in science contexts and to help identify some of the challenges experienced by students when learning science
- stimulate discussion and potential collaboration between groups of teachers either within a science department or between science teachers and others such as mathematics and geography teachers or senior leaders in the school.


## Teaching approaches

The eight accounts submitted by teachers are listed below.

- A summary of each account is provided in Section A: Commentaries.
- Full accounts are provided in Section B: Teacher accounts.

1. Cross-curricular approaches to graph drawing

A whole-school approach to concerns about children failing to apply what had been learned in their mathematics lessons to a science context. The students were aged 11 in this 11-18 rural school.
2. Deriving quantities from gradients

A focus on calculating the gradient of a graph and interpreting its meaning. The use of a spreadsheet is compared with manual methods of calculating gradients with 13 -year-old students in a co-educational independent school.
3. Using a literacy approach to interpreting graphs

These 11-year-old students in an urban secondary school had limited literacy skills, so their teacher used stories to illustrate the relationship between variables.
4. Introducing terms to describe data types

Typically some teaching sets of 11-year-olds in this 11-18 school are not taught about data types in mathematics lessons. This account describes collaboration between mathematics and science teachers.
5. Joint mathematics and science day to teach equations and graphs

A range of activities undertaken to consolidate learning for more able 13-year- old students in an urban secondary school.
6. The vocabulary of graphs - an example of departmental collaboration

Exploring the differences in language, and the types of graphs encountered by 14-yearold students.
7. Molar calculations in chemistry

Using similar approaches to those used in mathematics to calculate quantities, with 15 -year-old students in a grammar school.

## 8. Interpreting graphs

Improving high-attaining 14-year-old students' confidence in describing and explaining graphs at the start of their examination course.

The accounts appear in sequence to approximately cover a spectrum from whole-school, to small-scale collaborations and individual teacher approaches.

## Commentaries (Section A)

The purpose of the commentaries is to provide an overview of each account and to identify key issues. This may help the reader select specific material for further reading and provide a stimulus for reflection or discussion.

The commentaries comprise:

- the context of the account, including the science teaching context, aspects of mathematics that students find difficult, the age of the students and type of school
- a lesson summary, briefly describing the main activities
- selected illustrations from the lesson exemplifying the approach and/or outcomes
- a commentary on key points, to stimulate discussion or reflection
- links to relevant sections in The Language of Mathematics in Science: A Guide for Teachers of 11-16 Science
- broader points, which describe generalisable features that could be applied to other contexts
- questions for the reader, to prompt reflection
- postscript, in some cases, containing reflections of the author on the teaching episode.


## Teacher accounts (Section B)

Each teacher account appears in full. Where possible, accompanying illustrations of presentations used in the lesson, and other materials, including student responses, have been included.
A number of teachers volunteered to describe their experiences. These particular accounts were selected to illustrate different solutions across a range of contexts. Many of the accounts feature some aspect of drawing or interpreting graphs and this is no surprise - graphs are repeatedly identified by science teachers as causing problems for their students. What emerges is the need to clarify language used in mathematics and science lessons, the need for some common approaches to be adopted across the school, and clarification of the purposes and expectations of using particular procedures.
The accounts were written before The Language of Mathematics in Science: A Guide for Teachers of 11-16 Science had been published, and therefore the approaches described may not necessarily reflect the content of the guidance materials. The authors are listed in Acknowledgements below; the project team is grateful to each of them for 'lifting the lid' on their classroom practice and for providing these rich insights into the challenges and solutions involved in teaching science to youngsters in secondary schools.

## Acknowledgements

The project is indebted to the following exceptional teachers who provided insights into their classrooms:

- Mike Davies
- George Duckworth
- Louise Herbert with Nigel Atton
- Jon Hickman
- Mike Jackson
- Eliza McIntosh
- Alison Simpson
- Hannah Venning

Many other teachers have contributed to the project through workshops, discussion and feedback, including reviewing the emerging materials. Their contributions are much appreciated.

## Section A: Commentaries

## 1. Cross-curricular approaches to graph drawing

This commentary was written by the editor and draws on the text of the teacher's own account. It includes extracts from the account, which can be found on page 30 .

A science department recognised that students were failing to make good progress in their science lessons. They believed that a significant factor was a failure by students to apply the mathematical skills and understanding learned in mathematics lessons to a science-learning context.
Their initial response was to work with colleagues teaching mathematics to consider how the timing of teaching skills in mathematics coincided with teaching the same skills in science. A joint teaching resource on graph drawing was produced at this early stage of collaboration.
The collaboration has now developed into a whole-school initiative to consider numeracy across the curriculum. A coordinated approach, with individuals in each department taking responsibility for cross-curricular communication has followed. A mathematics toolkit has been produced and the school is now taking steps to embed these changes across all departments.
The school is an 11-18 school with a rural catchment area, and the work described here involved Year 7 (11-year-old) students.

## Lesson summaries

The change in practice took place over several lessons and included the following activities:

1. A small number of students worked with teachers from both faculties to create a 'graph drawing mat'. This was then used as guidance for other students. (See illustration 1).
2. Science teachers adopted a similar lesson-planning format to that used by mathematics teachers.
3. Mathematics teachers and science teachers taught linked lessons on data handling in the context of 'All about us'.
4. Science and mathematics teachers set joint homework to analyse data, using the context of 'Space'.

## Illustrations from full report

(The full report can be found in Section B on page 30.)
Illustration 1 An example of a learning mat on graph drawing produced by 11-year-old student


Illustration 2 Part of shared lesson plan format

## Year 7 Summer Term Variation and Inheritance

## Lessons 5 \& 6 More About Ourselves

## Learning Outcomes:

- Investigate the variation between individuals of the same species
- Describe variation between individuals as continuous or discontinuous
- Understand that characteristics can be inherited
- Understand that the environment can affect variation


## Numeracy Outcome

- Make measurements using appropriate units
- Collect data in appropriate tables
- Draw appropriate graphs


## Success Criteria:

- Work collaboratively to collect data on class in appropriate tables
- Draw appropriate graphs
- Describe your data as continuous or discontinuous
- Explain how variation within a species is due to genes, environment or both


## Commentary on key points

- Both mathematics teachers and science teachers recognised the benefits of collaborating to improve the transferability of mathematical skills. This evolved into a whole-school initiative aimed at improving numeracy (and literacy) across the curriculum.
- Differences in the language used in science and mathematics lessons were recognised at an early stage in the project and were identified as a cause of confusion for students.
- Mathematics and science teachers now use a common lesson plan format where a common mathematics focus is identified.

Synchronised lessons in mathematics and science on data handling have assisted students in making links between subjects. This has led to mathematics teachers using equations from science to teach algebra.
Each of these points reinforces the need for good communication between teachers, and a commitment to cross-curricular working. Given the variety of pressures encountered in schools such collaboration can be difficult to initiate and maintain, and requires effective leadership.

## Links to the Guide

Many of the words and ideas featured in this account are explained throughout The Language of Mathematics in Science: A Guide for Teachers of 11-16 Science, and specifically in the following sections:

- Chapter 4 Drawing charts and graphs for choosing scales and plotting points
- Section 7.4 Lines of best fit
- Sections 9.6 and 9.7 Rearranging equations - the two sections deal separately with equations that involve addition or subtraction, and those that involve multiplication and division.


## Broader points arising

Discussions with colleagues revealed that rearranging equations, drawing and interpreting graphs and understanding different types of graph were particular issues for their students. The areas of mathematical challenge at this school are similar to those identified by other teachers who contributed to The Language of Mathematics in Science project. In this case the solution was to highlight numeracy (and literacy) issues across the curriculum, with a particular focus on raising the attainment of low attaining students. This required the creation of new roles for some staff, and the adoption of a longterm strategy.

## Prompting questions for readers

- Do your colleagues have an agreed approach on how to deal with 'problem areas' like those mentioned here (adding lines of best fit to points on a graph, rearranging equations, and dealing with anomalous measurement data points)?
- Does the appointment of a numeracy coordinator within a department have the unintended effect of removing a sense of responsibility for mathematical issues from other teachers?


## Postscript

Further thoughts from the author:
Numeracy has remained a high profile issue at our school. In science it is recognised by all that a student's numeracy skills in science must be developed alongside the skills they learn in maths. This raised awareness is the greatest outcome of our work on numeracy so far. Our progress seems very small, however, when the challenges of the curriculum changes are taken into account. The level of demand has been raised at $A$ level and in the new GCSE. The challenge now is to integrate teaching the maths skills in science from KS3. Collaboration between faculties together with continued appreciation of the student's own experiences will be increasingly important if we are to succeed in raising the levels of numeracy in science into the future.

## 2. Deriving quantities from gradients

This commentary was written by the editor and draws on the text of the teacher's own account. It includes extracts from the account, which can be found on page 37 .

This account describes a lesson aimed at addressing problems students have in interpreting graphs. The lesson was taught to different groups so that the use of computer generated graphs could be compared to using hand drawn graphs. Evidence was collected to measure the impact on students' ability to use the gradient of a graph to find a quantity.
The students were Year 9 (13-14 years old) and they were studying for IGCSE in physics. The school is a co-educational independent school with a higher than average attainment level as measured by nationally available attainment prediction tests.

## Lesson summary

The context for this lesson was exploring the relationship between load and extension of a spring.
Three classes of students were given a questionnaire to explore their ability to find the gradient of a graph and to use this to explain the relationship between different variables.
The classes then carried out an activity on measuring the extension of a spring under different loads, as part of their work on Hooke's Law. They plotted their data on a suitable graph to show the relationship between the two variables and were then asked to calculate the gradient of a line of best fit, and to find the constant of the spring.
They then completed a second questionnaire.
One of the classes was asked to draw graphs by hand, one was asked to use a spreadsheet to draw the graph; the third class was asked to use both methods. Many of the students were unfamiliar with using spreadsheets in science lessons, and were provided with guidance materials.

## Illustrations from full report

(The full report can be found in Section B on page 37.)

Illustration 1 Part of the questionnaire used before the lesson on spring extension

1. Rearrange the equation $x=y z$ to make $y$ the subject
2. Rearrange the equation $F=k x$ to make $x$ the subject
3. Rearrange the equation $I=V / R$ to make $R$ the subject
4. What is meant by the gradient of a line?
5. Calculate the gradient of a line which passes through $(4,2)$ and $(8,4)$
6. Calculate the gradient of the graph below

7. What does it mean if two variables are directly proportional?
8. If $F$ is directly proportional to $a$ with a constant of proportionality $m$, write an equation linking $F, m$ and $a$.

Illustration 2 Part of worksheet used in lesson on spring extension
Table of Results:

| Mass on spring <br> $\mathrm{m} /$ | Force on spring <br> F / | Stretched length <br> I/ | Extension of spring x/ |
| :---: | :---: | :---: | :---: |
| $\mathbf{1 0 0}$ |  |  |  |
|  |  |  |  |
|  |  |  |  |

## You should all....

1. On graph paper or using Excel, plot a graph of Force against Extension. Label the axes and units and draw a line of best fit for the points. If you use Excel, print the graph off and draw the line of best fit by hand.
2. Describe the relationship between Force and Extension for your spring. Justify your answer by reference to your graph.
3. Calculate the spring constant for your spring and label the limit of proportionality (if it has been reached)
4. Describe how you ensured that your measurements were accurate.

## Most of you will also...

1. Using your graph (or otherwise) make predictions of lengths and extensions for values in the table below

## Commentary on key points

Most, but not all, students in this study were able to draw an appropriate line of best fit on the graphs drawn from their own data. They were less successful at being able to use the gradient of this line to calculate the spring constant, preferring to calculate the value from data in their results tables. This suggests that they did not recognise the importance of using a graph in data analysis. The mathematical use of gradients needs to be firmly embedded in science teaching in a range of contexts, particularly because recent science specification changes require students to measure the gradient of tangents to curves as an indication of reaction rate.
The use of spreadsheets to draw graphs has been the subject of discussion amongst science teachers in recent years. In this study, there was an opportunity to use the trend line function to speed up and simplify the calculation of gradients, yet many students preferred to use manual methods as they lacked familiarity and found using spreadsheets confusing. At what age should science teachers be teaching their classes to use scientific calculators and spreadsheets?

## Links to the Guide

Many of the words and ideas featured in this account are explained throughout The Language of Mathematics in Science: A Guide for Teachers of 11-16 Science, and specifically in the following section:

- Chapter 5 (Working with proportionality and ratio) explains how graphs can be used to represent the relationship between variables that are proportional to each other. The chapter goes on to show how to find the gradient of a graph and how the gradient can be used to find the constant of proportionality in an equation.


## Broader points arising

This study considered proportionality and the use of gradients to calculate quantities. The teacher had checked that these topics had already been taught to the students in mathematics lessons, and yet many of the students were unable to apply their mathematics learning to a science context. One reason for this is that what is taught in mathematics lessons may have differences in purpose or approach. For example, graphs are rarely used in mathematics lessons to explore the relationships between variables. The Language of Mathematics in Science guidance book aims to help science teachers recognise the approaches used by mathematics colleagues, but there is also a need for good communication and collaboration between the two subjects.
The nature and purpose of learning mathematics is different for mathematicians and scientists. This study illustrated some of these difficulties:

- Students who can rearrange equations of the form $x=y z$ may not be able to rearrange similar equations in the form $F=k x$. In mathematics the symbols in an equation represent a number, whereas in science symbols such as $F, m$ and $a$ represent specific quantities and have units.
- Similarly the gradient of a line on a graph represents a ratio in mathematics, but it represents a quantity in science and therefore units must be included.


## Prompting questions for readers

- What explanation of the term 'direct proportionality' would you find acceptable from Year 9 students?
- How would you modify the approach used here to help lower attaining students understand how to use graphs to explore relationships between variables?
- What are your views on the use of spreadsheets to support learning in contexts that use graphs to explore relationships between variables?


## Postscript

Further thoughts and reflections contributed by the author:
It is clear that both the mathematical ability and the ability of students to make links between concepts studied in maths lessons and scientific data can be extremely varied. Once the experiment and questionnaire for the study, focusing on assessing students' current level of understanding and where misconceptions arose, had been completed the key concepts were recapped both briefly in a whole-class setting and in more detail with those individuals who had struggled the most. Students certainly found these discussions useful and many declared that it had begun to make sense to them. Future good practice could include giving students a questionnaire similar to that given at the start of the project to assess their mathematical understanding in preparation for any experimental work on proportionality, with time then allowed to give students feedback on the questions from the questionnaire and ensure concepts are clear before starting on the new physics. As discussed in the main report, more frequent use of short mathematical starter activities at the start of lessons relating to the physics to be covered can also help students to use maths in their science.

## 3. Using a literacy approach to interpreting graphs

This commentary was written by the editor and draws on the text of the teacher's own account. It includes extracts from the account, which can be found on page 51.

This teaching account describes a single science lesson designed to introduce students to the mathematical skills involved in interpreting graphs, before students are taught how to draw graphs of their own data. The approach used focuses on the use of language to describe relationships between variables.

The students whose work is described here come from an economically deprived area and many have weak literacy and numeracy skills. In this class of 11-year-olds more than half the students have some form of SEN provision.

## Lesson summary

The lesson aimed to help students appreciate that the axes of a graph are linked, reflecting the relationship between two variables. This consisted of several stages:

1. Students were asked to describe graphs using 'scientific sentences' of the form 'The higher the ..., the higher (or lower) the...'
2. Translating the position of a line into the quantities being represented. This involved using sentences of the form 'The more the $\ldots$, the more (or less) the ...'
3. Introducing the idea that the gradient of a graph is an indication of a rate of change in the variables.
4. Asking students to generate sketch graphs to illustrate aspects of a story that had been read to them. The story was Little Red Riding Hood and the graph they sketched was to illustrate how the lead character's happiness changed over time. They then went on to tell their own stories as graphs.

## Illustrations from full report

(The full report can be found in Section B on page 51.)

Illustration 1 Exploring the relationship between the two variables
Figure 1 shows an example of an activity intended to reinforce the link between the graphical and linguistic. The student wrote 'The higher the average wind speed is the lower the rate of erosion'. The teacher corrected this to say 'The higher the average wind speed, the higher the rate of erosion'. This topic had recently been studied in geography lessons.


Figure 1

Illustration 2 Students creating their own graphs from stories
Figure 2 shows a graph created by a student to represent the change in pain levels felt in his leg when he fell from a bike and then his mother put a dressing over his grazed knee.


Figure 2

## Commentary on key points

The teacher has based their approach to understanding graphs on a literacy model developed by Paolo Freire in Brazil in the 1970s.

Students' initial analysis of graphs is qualitative only and emphasises the idea of graphs 'telling a story'.
The skills of interpreting graphs have been broken down into a series of distinct stages. Novel learning activities using a broad range of curricular contexts have been planned for each stage, with the intention to move students on in their understanding towards a clearly defined goal. Student responses to these activities are used to identify areas of weakness and where further work is needed.

## Links to the Guide

Many of the words and ideas featured in this account are explained throughout The Language of Mathematics in Science: A Guide for Teachers of 11-16 Science, and specifically in the following sections:

- Section 7.1 Types of relationship and shapes of line graphs
- Section 7.2 Developing a descriptive language
- Section 7.3 Gradients and rates of change


## Broader points arising

In mathematics lessons students do not tend to consider the relationship between the variables represented by each axis of a graph, yet in science lessons this is an essential part of analysing data. The account shows that students need considerable support in their science lessons to be able to interpret graphs to further their scientific understanding. In particular, interpreting the gradient of the graph presents considerable challenge, a skill featured in account 2 (Deriving quantities from gradients).
Beyond science, it is useful for citizens to be aware of the implications of different ways that information can be represented. A particular difficulty for many people is understanding the difference between correlation and causation. This teacher's philosophy is to prepare students for life beyond school, as well as provide them with the essential tools to succeed in learning science.

## Prompting questions for readers

- In this account a number of different scaffolding techniques were used such as writing frames for sentences. The students found difficulty once the scaffolding was removed. How could you help students make the transition from fully scaffolded approaches to independent working?
- One of the constraints of this article is that only written responses can be shown. How would you plan to use dialogue to prepare students before they made a written response?
- What further activities would you plan for students who through their written responses showed that they were unable to relate the steepness of a graph to a rate of change?


## Postscript

The author made the following comment several months after submitting their account.
Developing the toolkit to interpret graphs has been invaluable in teaching not only lower ability students but also top set students who look even deeper into graphs and use the same methods to unpick the meaning that underlies the data. The most powerful aspect, especially with lower ability groups, has been the more/more, more/same and morelless structure, which allows students to easily translate the shape of a graph into a sentence that describes it.

## 4. Introducing terms used to describe data types

This commentary was written by the editor and draws on the text of the teacher's own account. It includes extracts from the account, which can be found on page 57 .

The account describes an approach used by a science teacher to introduce language that distinguishes between different types of data encountered in practical science. Students were successful in applying these terms in the context of this lesson, but many could not then apply the terms to new contexts. The author was a PGCE student at the time the account was produced, and the class comprised 28 11-year-old students.

## Lesson summary

The lesson aim was for students to be able to confidently display their experimental results using correct conventions of school science. The context used was measuring the temperature change of water when heated by different fuels.

1. Students read instructions for practical work and then answer questions on different types of variable (dependent, independent, and control variables).
2. Teacher explanation of quantitative, qualitative, continuous and discontinuous data.
3. Students carry out practical work in pairs.
4. Students complete a task sheet. Responses analysed by the teacher to gauge success of the lesson.

## Illustrations from full report

(The full report can be found in Section B on page 57.)

## Illustration 1 Terms used to describe variables and data

## Language of maths: collecting data

Different ways of collecting data - counting and measuring qualitative and quantitative data. Measuring instruments and scales. Collecting sets of data - variables - different types of data.

Key words: quantity, value, unit, quantitative data, qualitative data, significant figures, variable, continuous data, discrete data, categorical data, time series, grouped data, primary data, secondary data.

Figure 1 introductory slide from presentation used in lesson

## Illustration 2 Definition of terms

| Key Word | Definition |
| :--- | :--- |
|  | The thing YOU change |
|  | The thing you are measuring |
|  | What you keep the same to make it a fair <br> test |
| Discrete data | Data that can only take certain values |
| Primary data | Data you have taken yourself |
| Secondary data | Data you have taken from someone else |
| Qualitative data | Descriptions (results are words) |
| Quantitative data | Counting (results are numbers) |
| Continuous | Data that can take any value (within a <br> range) |
|  |  |

Figure 2 Definitions slide from presentation used in lesson

Illustration 3 Task sheet


I sampled how long everyone's hair was in the staffroom. Is this continuous or discrete data?

Figure 3 Task sheet completed by students following practical activity
This sheet was used as continual assessment during the lesson and sheets were marked afterwards. Fourteen students completed everything on the sheet correctly, including the extension.

## Commentary on key points

The context for this activity was a practical investigation but one that focused on measurement and data rather than the learning of challenging practical techniques or difficult science concepts. The choice of appropriate practical activities to use for a particular learning outcome is crucial if students are to achieve the specific purpose of the activity. In this activity the teacher was careful to choose a context that was interesting and exciting for children as it was recognised that children often associate learning with a particular activity. The context used in this case was using burners with different fuels to heat water, which the teacher believed would stay in students' memories for considerably longer than if they had simply described the meaning of the terms.
In discussion with mathematics colleagues, the teacher found that only the top mathematics teaching sets would have been familiar with the use of these data terms. This is quite a common situation where children in science groups may be taught in different sets for mathematics, and will not all have been taught the same mathematical skills and ideas.
The account raises the question of how best to support language acquisition in children. Is it more effective to teach new vocabulary and then select the most appropriate labels to use in some further activity, such investigating fuels as described here? An alternative approach would be to analyse and group data from a practical experience, to compare and contrast data types and then to introduce the correct terms.

## Links to the Guide

Many of the words and ideas featured in this account are explained throughout The Language of Mathematics in Science: A Guide for Teachers of 11-16 Science, and specifically in the following sections:

- Section 1.3 Characteristics of different types of data
- Section 1.4 Naming different types of data
- Section 1.5 Where do data come from?

This account was written before the guidance materials were made available, and so there may be variances in terminology described here, and in the guidance materials.

## Broader points arising

This teacher's own lack of personal confidence with mathematics from their own school days meant that they understood the challenges that many students face. Those who had not previously been introduced to the data terms were carefully supported until they could use the terms with confidence. This led to a recognition of the general importance of working collaboratively with colleagues in the mathematics department and making sure that science teachers are aware of which students have already encountered different mathematical ideas.
This teacher identified the retention of information and the ability to apply as two different aspects of student learning. It was found that most of the students in the class could correctly retain information in the context of the experiment but half of them had difficulty in applying it to other contexts. Acquiring new ideas and language take time and practice. In the conclusion, the teacher recommended that further activities and dialogue were needed in order to embed learning and enable students to apply the meaning of these terms outside their direct experience.

## Prompting questions for readers

- The language of measurement in science guidance book uses the term discrete to describe numerical data that can only take on certain values. In this report both discrete and discontinuous are used as synonymous terms. What terminology do mathematics and science teachers in your establishment use?
- What further activities would you recommend to the teacher, to support students who find it difficult on the task sheet to distinguish between continuous and discrete data types?
- The author of this account wrote:
'It really struck me during the experience how little the two departments talk and so just a few links embedded in lessons and an awareness of subject overlap could be a really powerful tool.' How frequently do teachers of mathematics and science communicate about learning in your school and what could you do to ensure that mathematics links could be embedded in science lessons?


## Postscript

Further thoughts and reflections contributed by the author:
I would definitely use this approach again as I think it helped them to see the bigger picture of their investigation. I would probably do this over two lessons - 1 lesson for planning then consolidate this knowledge in the next lesson before practical then conclude. I think the downside is that there was a lot of information for them to take in so doing it over 2 lessons would be better. Additionally, I now use investigation sheets with similar questions for all of my Year 7 and 8 lessons to get them used to using the language so when they reach GCSE/A-Level it is already in their brains.
I stand by my comment that maths and science need to work together more closely - it is easy for the same vocabulary to be used in both lessons. By both sides using the same words over and over, in the correct context (!) students will become confident in how to use it. I think also that by using it in different contexts - for example using science examples within a maths classroom means that when students are faced with data that they haven't seen before, they have already practiced applying the skills in different classrooms/contexts/situations.

## 5. Joint mathematics and science day to teach equations and graphs

This commentary was written by the editor and draws on the text of the teacher's own account. It includes extracts from the account, which can be found on page 63.

A day off timetable was organised for a group of 60 students. A science teacher and two mathematics colleagues aimed to teach mathematical skills associated with motion. The context chosen was around cars and car chases as this allowed concepts of speed, velocity, acceleration, distance, time and kinetic energy to be explored. These concepts form an important part of both the mathematics and science curricula.

This account describes some of the activities planned through this collaboration and how groups of students responded.
All the students were in one of the top two sets for mathematics in Year 9 aged between 13-14 years old.

## Lesson summary

The mathematical content for the day was:

- using a formula to calculate values for the kinetic energy of a car travelling at different speeds;
- rearranging the speed-distance-time formula to make different variables the subject of the formula;
- drawing graphs of speed against kinetic energy;
- interpreting graphs.

To consolidate learning, groups were set an application task that required them to analyse clips from feature films.

## Illustration from full report

(The full report can be found in Section B on page 63.)

## Application task - analysing feature film clips

Groups of students were given the titles of four films featuring car chases: Bullitt, The Bourne Identity, Quantum of Solace and The Italian Job.
After watching the clips online, students plotted the route of each car chase using mapping software to calculate the distance travelled. They were then able to estimate the average speed for each journey.


## Transcript

We think the Quantum of Solace car chase is the least realistic car chase as it is 19 times faster than the fastest speed ever recorded by a car. This is in comparison to Bullitt (which is under the fastest speed) and Bourne Identity (which is 3 times faster than the fastest speed)

## Slide from a student presentation explaining their analysis of car chases featured in different films

## Commentary on key points

The activity targeted a specific group of Year 9 students, and was planned for a single day, when the normal school timetable was suspended for these teachers and students. The challenge will be how to apply these outcomes across the school, to have influence on all students, and to find ways for mathematics and science teachers to collaborate on all areas where there is curriculum overlap.
The teachers introduced the work on rearranging formulae by using a triangle method. This allows students to find the subject of a formula by covering a symbol within the triangle diagram and using the position of the other two symbols to decide which mathematical operator applies.
For example, the formula for calculating speed is to divide distance by time:

$$
\text { Speed }=\frac{\text { Distance }}{\text { Time }}
$$

The triangle diagram used was:


To make Distance the subject of the formula, 'Distance' in the triangle is covered up. This leaves 'Speed $\times$ Time' visible so that:

$$
\text { Distance }=\text { Speed } \times \text { Time }
$$

Some teachers are against the use of such a technique, and comment that it prevents students from learning how to rearrange equations 'correctly' through the use of mathematics. In this instance it was interesting that in this school students were taught the triangle method in their mathematics lessons. Also these teachers commented that rearranging formulae using the triangle method is an 'easy starting point'. Other teachers prefer to use the triangle method only after the students have already been taught to rearrange formulae mathematically.

## Links to the Guide

Many of the words and ideas featured in this account are explained throughout The Language of Mathematics in Science: A Guide for Teachers of 11-16 Science, and specifically in the following sections:

- Section 9.5 The real-world meaning of a formula
- Section 9.7 Rearranging equations involving multiplication and division
- Section 9.10 Use of 'calculation triangles'
- Section 9.11 Mathematical equations and relationships in science

This report was written before the guidance materials were made available, and so there may be variances in terminology described in this account, and in the guidance materials.

## Broader points arising

The science teacher described this teaching event as a successful collaboration. By having mathematics and science teachers present it helped to emphasise to students who took part that the skills they had learned were applicable to both subjects. Also having a similar approach to teaching new skills, the two departments helped students to transfer these skills between subjects. There were two important outcomes from this intervention:

- students benefitted by recognising skills learned in one subject could be applied elsewhere;
- teachers understood the importance of having common approaches to teaching.


## Prompting questions for readers

- A number of online tools such as maps and simulations provide rich contexts and support for teaching mathematical skills. Others, such as spreadsheets and geometry software have been designed specifically for mathematical operations. What is your assessment of the potential for the use of such resources in your own teaching?
- What stimulus would trigger closer collaboration between science and mathematics teachers in your school?
- Should science teachers be teaching their students better mathematical understanding rather than teaching just the mathematical procedures needed to achieve required solutions to problems?


## 6. The vocabulary of graphs - an example of departmental collaboration

This commentary was written by the editor and draws on the text of the teacher's own account. It includes extracts from the account, which can be found on page 70.

Difficulties students experienced in analysing graphs to identify relationships between variables led to their teachers recognising that language was being used differently in mathematics and science lessons. This created a better understanding of the differences between the mathematics taught in science lessons and that taught in mathematics lessons. Collaboration between the science teacher and a mathematics colleague resulted in the production of teaching activities for science and a longer-term target to modify schemes of learning in mathematics.
The teachers involved in this approach are lead practitioners in science and mathematics at an 11-18 comprehensive school. The students involved were 14 -year-olds in the middle set for science.
The account illustrates the way that experienced science and mathematics teachers have worked together to improve student learning in both subjects, by focusing on the meaning of key words.

## Lesson summary

Create 'baseline graphs' of data sets


Identify key terminology and discuss the various meanings of the words held by the class


Apply newly created meanings of key words to a range of example graphs


Students independently apply key words to their baseline graph and generate their own conclusions


Class decides on the best student examples and uses these to create a classroom display

## Illustrations from full report

Indented text indicates an extract taken from the teacher account published in full in Section B on page 70 .

## Extract 1

Discussion between science and mathematics teachers revealed a number of differences in the way that mathematics was used and described when constructing and analysing graphs.

> In discussion between the two teachers involved we also identified that: (a) in maths lessons they only ever drew straight lines of best fit and looked at linear relationships, leading students to do this in science; (b) maths teachers very rarely commented on the relationships between variables and did not use the word variables often; (c) in maths ranges were only used in data and never to describe the range of values chosen for a variable.

## Extract 2

The class created their own definitions of key words through discussion around activities, rather than accepting definitions from authoritative external sources.

We then identified the 4 most important words they needed as being correlation, line of best fit, pattern and range. We sorted out meanings that were the best from a set of possible true or false statements, for example 'to have a pattern from a set of data, we must be able to see it goes up in even amounts e.g. in 10s' or 'a positive correlation must always be a straight line going up'.

## Commentary on key points

The authors identified a number of areas where terminology in mathematics teaching differs from that used in science lessons:

- 'line of best fit' is always a (straight) line in mathematics, but can also be a curve in science;
- 'variable' is used infrequently in mathematics, but very commonly used in science, with students being expected to identify different categories of variable by age 11 ;
- 'range' is a numerical value in mathematics, but a quantity in science, linked to a specific variable.

There are a number of other cases where the use of language differs in mathematics and science.
Further examples are given in the The Language of Mathematics in Science guidance, but as this account shows, dialogue between mathematics and science colleagues can be effective in identifying some of these differences and then agreeing on how best to support students.
Discussing the meaning of key words used when describing graphs revealed a number of alternative meanings held by students, and also helped the teacher recognise that the students' understanding of 'correlation' was inadequate for the intended activity. The students were unable to relate examples used in science to ideas of correlation used in mathematics:

## I thought you were talking about different things

Structured discussion as described in the account helped students to make links between the two subjects:

## I didn't know maths were asking us to plot graphs of actual experiments

This discussion helped move their understanding of key words towards a common meaning shared by mathematics and science.

## Links to the Guide

Many of the words and ideas featured in this account are explained throughout The Language of Mathematics in Science: A Guide for Teachers of 11-16 Science, and specifically in the following sections:

- Section 7.1 Types of relationship and shapes of line graphs
- Section 8.7 Relationships between variables: scatter graphs and correlation
- Section 8.8 Drawing a line of best fit on a scatter graph

As the account demonstrates, teachers need to use their professional judgement when deciding which ideas and definitions of words to use with particular groups of students.

## Broader points arising

Recognising the problems students experienced when transferring their mathematical skills between subjects was the first step that then brought about collaboration between the two departments. The teachers directly involved in the collaboration were both 'lead practitioners' and therefore had sufficient status within the school to bring about both immediate and long-term changes. In other situations, a science teacher may identify similar problems, but be unable to bring about changes to the curriculum without the support of middle and senior leaders.
There are many examples in science teaching where students need to be able to recognise that words with an everyday meaning may have quite a specific or different meaning in a science-teaching context. Words such as 'food', 'energy' and 'force' are familiar to science teachers as having different meanings. This account has shown that we also need to make students aware that words used in mathematics, such as 'line' and 'range', also have a different meaning when used in a science context. The account describes a strategy for helping students develop awareness of these different meanings, and to establish a 'shared meaning' for the class.

## Prompting questions for readers

- Literacy and numeracy initiatives in schools are often seen as addressing different problems. This account has shown that vocabulary plays a significant role in enabling students to apply their mathematical skills. What opportunities exist in your school for taking a combined approach to literacy and numeracy across the curriculum?
- What are the key points in the science curriculum where you would allocate lesson time to address the meaning of mathematical terms and vocabulary?
- Has your science department listed key vocabulary that has different meanings outside a scienceteaching context?


## 7. Molar calculations in chemistry

This commentary was written by the editor and draws on the text of the teacher's own account. It includes extracts from the account, which can be found on page 72 .

This account describes how a science teacher adopted exactly the same approaches and methods in science as students had experienced in their mathematics lessons. The change enabled a group of chemistry students to master 'reacting masses' calculations.
GCSE chemistry students need to be able to calculate the mass of product that can be produced from a given mass of reactant. Traditionally this has been taught alongside the use of the 'mole' concept and the use of chemical equations to state the proportions in which materials react. Finding that even the most able students struggle to use ideas of proportionality taught in mathematics in the context of molar calculations, a change of approach was tried.
The students were taught chemical calculations as part of their GCSE chemistry course when 14-15 years old, having been taught proportionality in mathematics when 12-13.

## Lesson summary

Previously, when teaching this topic, the following approach was used:

1. Using the mass of reactant to calculate the number of moles of reactant;
2. Using a balanced chemical equation to find the number of moles of product that can be formed;
3. Using this number of moles of product to calculate the mass of product formed.

Students struggled to use the mathematical formulae involved leading to problems in steps 1 and 3. They also found step 2 difficult, as they did not have a secure grasp of the significance of numbers used in a balanced chemical equation.
In this lesson, the detailed discussion of the mole concept was deferred, and teaching concentrated in helping students make links between chemical calculations and the methods they had been taught in mathematics. The students were told simply that the mole is an expression of the relative mass of a substance expressed in grams. A more detailed teaching of the nature of the mole was returned to once the chemical calculations had been mastered.

## Illustrations from full report

The full account of this teaching episode is given in Section B on page 72.

Extract 1 Finding the relative formula mass of an element given the mass of a compound The table illustrates the method used both in this chemistry lesson and by mathematics teachers at the school to calculate the mass of carbon in 300 g of carbon dioxide.
$\mathrm{CO}_{2}$-percentage masses of elements


## Extract 2

The method used for calculating the maximum mass of carbon dioxide produced by combustion of 100 g of ethane and the maximum mass of water produced by combustion of 25 kg of ethane.

The combustion of ethane


| Mass $\mathrm{C}_{2} \mathrm{H}_{6} / 8$ | Mass $\mathrm{O}_{2} / 8$ |  | Mass $\mathrm{CO}_{2} / 8$ | Mass $\mathrm{H}_{2} \mathrm{O} / 8$ |
| :--- | :--- | :--- | :--- | :--- |
| 100 | Excess | $\frac{176}{60} \times 100=293$ |  |  |
| 25000 | Excess |  |  | $\frac{108}{60} \times 25000=45000$ |

## Commentary on key points

Analysing the problems experienced by students led to breaking down learning into a series of discrete tasks. Ensuring that students could confidently calculate reacting masses before being introduced to the mole concept appears to have been successful in terms of mastering the calculations, although it will take some time for the full impact to be assessed. The account does not describe the impact on understanding the mole concept.

The success of this approach depended on exactly copying the techniques used by the mathematics department when teaching proportionality. This requires good communication and cooperation between colleagues, plus a clear understanding of the timing of different components of the mathematics curriculum in relation to the science curriculum.

Science teachers had previously provided their mathematics colleagues with examples of calculations carried out in chemistry lessons, so that the same examples could be used to contextualise mathematics teaching.

## Links to the Guide

Many of the words and ideas featured in this account are explained throughout The Language of Mathematics in Science: A Guide for Teachers of 11-16 Science, and specifically in the following sections:

- Section 5.6 Ratios
- Section 5.7 Proportional reasoning and ratios


## Broader points arising

This account illustrates a simple teaching point, but one that is often overlooked in a crowded curriculum - when teaching difficult procedures make sure that the context is either familiar or straightforward. Proportionality is often reported as presenting problems for students so removing the added complexity of understanding the mole concept enabled the students to master these challenging calculations.

Science teaching generates a wide range of data and situations that could be useful to mathematics teachers trying to teach their subject in contexts familiar to students. Schools are finding that closer collaboration between science and mathematics teachers is helping to break down the 'compartmentalisation' of subjects in the minds of their students.

## Prompting questions for readers

- Ideas about proportionality are used when teaching about transformers, monohybrid inheritance and chromatography. Think about other topics that draw on proportionality ideas, and decide how you could help students to make links between them.
- What examples of data and contexts could you share with mathematics colleagues to support their teaching, and thereby support the development of mathematical skills of your science students?


## 8. Interpreting graphs

This commentary was written by the editor and draws on the text of the teacher's own account. It includes extracts from the account, which can be found on page 75 .

The account outlines a particular science lesson in which students were asked to describe, explain and analyse information displayed in a challenging and novel graph. Confidence grids were used by students pre and post lesson to self-report on a number of increasingly complex tasks.

In the past students at the school have had difficulties when tackling examination questions that asked them to 'describe' and 'explain' graphs.
The students in this study were aged 14 and about to start their GCSE courses. They were girls in a high ability class.

## Lesson summary

The context of the lesson was a graph representing the speed of Felix Baumgartner in a skydive that took place in October 2012. The graph also shows the speed of sound at different points throughout Baumgartner's descent as a separate curve.

Students were asked to assess their confidence on a three point scale against a number of statements relating to the interpretation of information in this graph.
They then completed a number of activities before being asked to complete the confidence grid once more.

The activities were:

1. Describe a different graph to a partner who then had to sketch the graph without seeing the original.
2. Explain information in the Baumgartner graph such as speed at particular times in the descent.
3. Analyse information by making mathematical comparisons of values at specific points. Students were also asked to calculate distance by estimating area under the graph curve.
4. Link information in the graph to ideas about forces.
5. Create a further graph from data derived from the first graph (speed of the skydiver in relation to the speed of sound)

## Illustrations from full report

The full account of this teaching episode is given in Section B on page 75. It includes the graph used as the lesson context, and a summary of the confidence grid responses.

## Commentary on key points

The lesson resulted in improved confidence in describing graphs (as reported in the confidence grids) and to a lesser extent improved confidence in explaining graphs. It appears not all students completed the more demanding activities, and that a second lesson would be required to achieve all the intended learning outcomes.
Using a confidence grid at the start of the lesson saved time, as information was used to modify lesson plans and avoid the unnecessary teaching of familiar skills. The teacher made the point that further checks could have been employed to make sure the self-reporting was accurate. Such a system of informal assessment of key skills could be further developed to provide effective feedback to the teacher without necessarily using up a lot of lesson time.

Science teachers in this school used a 'hierarchy' of command words for structuring their teaching, using the acronym 'IDEALS' (Identify, Describe, Explain, Analyse, Link, Synthesise) to represent increasing levels of demand on learners. The lesson summary provides an example of how this system was applied in the graph interpretation lesson. Although the outcomes of the post lesson confidence grid suggest that more students were able to describe than explain, this was not always the case. The results raise the question of whether such a hierarchy is appropriate in all contexts for all students.

## Links to the Guide

Many of the words and ideas featured in this account are explained throughout The Language of Mathematics in Science: A Guide for Teachers of 11-16 Science, and specifically in the following sections:

- Section 7.2 Developing a descriptive language helping students to relate the shape of a graph to the meaning of the relationship between variables.
- Section 9.12 Graphs of quantities against time: gradients
- Section 9.13 Graphs of rates against time: area under the line
- Section 10.6 Movement of objects: speed, and velocity


## Broader points arising

One of the students commented in their questionnaire response:
It improved my confidence but also helped me develop skills I had forgotten.
This illustrates some of the complexities of learning science alongside other subjects in the curriculum - there is a lot to learn and remember when you are a school student. Reminding students of the skills required to complete a lesson task is helpful, and enables students to transfer their skills to new contexts, which in this case was calculating a distance travelled by estimating the area under the curve of a velocity / time graph. The teacher suggested linking area under a graph to the mathematical skills of calculating the area of triangles and estimating area. For this to be successful the school would need to build such skills into their science teaching at an earlier age.

At several points in the account the teacher suggests improvements to the lesson design including providing further activities with different data to help secure students' confidence. Using a graph of Felix Baumgartner's famous skydive provided a novel and interesting context. A bank of resources that provides such contexts would be a useful asset to a department, and yet appropriate curriculum and age related material for developing mathematical skills is not yet available in all schools.

## Prompting questions for readers

- What proportion of science lessons should be allocated to the development of mathematical skills (such as graph interpretation) alongside developing conceptual understanding of science, and the development of practical techniques?
- The account mentions providing exemplar material for students lacking confidence in their mathematical skills. Would the inclusion of peer assessment in lesson activities provide an effective alternative to 'exemplar material'?


## Postscript by teacher

The activity was useful in developing understanding of the finer details of students' abilities and areas of challenge. For instance they could describe a graph easily but found it difficult to explain what the graph meant in some instances. In light of the experience I would be less inclined (with students of the same ability) to be concerned with making too many links to other topics and would spend more time repeating and developing skills, allowing more time for interventions with those lacking confidence (andlor ability to progress). I will also be building this approach in to shorter and more regular tasks in this area to ensure frequent and varied approaches to the skills with sufficient time to ensure all students are developing.

## Section B: Teacher accounts

# 1. Cross-curricular approaches to graph drawing 

This is an account written directly by a teacher; a commentary by the editor is provided on page 2 .

The Science Faculty has been working to form closer links with the Maths Faculty. This was part of a whole-school improvement plan initiative to raise standards in numeracy but has provided the impetus for us to address problems with the language of maths that were already recognised within science. This is a case study of our work so far and it is very much a work in progress. We have identified a problem and we are developing working solutions but seeing positive outcomes is a long term process.

The initial stimulus for collaborative work between the two faculties came from science teachers who identified that there were numeracy problems preventing students making rapid progress in science. Skills that should be transferable were not being applied by students outside of maths lessons. These were principally related to: changing the subject in equations; scaling graphs; drawing lines of best fit; and identifying types of graphs e.g. scatter or line? The problems were confounded by a mismatch in the order in which certain maths skills were taught in KS3 and applied in science. These timing issues meant that in science we were expecting high level skills that had not yet been fully covered in maths.
We set out to address graph drawing, by collaborative work with maths to produce a teaching resource, but quickly the project evolved into a change in approach to numeracy across the entire science curriculum. Our aim is to raise the level of numeracy of all our students, to help them to recognise that skills can be transferred between subjects within school and then applied to new contexts in adult life.
The incentives for change are numerous. At a faculty level, for both science and maths, the incentives are better outcomes for our students, particularly our lower ability students. If mathematical questions are made accessible to students their overall attainment will be raised. At a whole-school level we can demonstrate to students that the skills they learn in maths are transferable. If students see that their maths skills can be applied in science then they should see that they can be applied to other subjects too. This recognition that skills are transferable and can be applied to solving problems in completely new situations should help our students take maths out into everyday life. We want our students to be equipped with the tools to question data throughout their lives.
The first challenge to sustained change is to raise numeracy as an issue across the entire school curriculum. This has now been done. The second is to co-ordinate the approach. Maths has a member of staff with responsibility for numeracy across the school and in science we have a member of staff with responsibility to collaborate with other faculties. A concrete outcome from our collaborative work has been a maths 'toolkit', a series of documents available to all staff on areas such as percentage
change, ratio and proportion, histograms and exponentials to support non maths specialists in teaching these topics within their subject areas. The third is to ensure that any changes are firmly embedded within the curriculum. This is achieved by writing numeracy and literacy learning goals into all schemes of work so that the links are constantly reinforced. It has to be recognised, however, that even with enhanced maths skills students' levels of literacy can still be a limiting factor to progress. We are therefore trying to develop an action plan for literacy parallel to that for numeracy.
It is difficult to assess the success of this project at this early stage, but there are some concrete outcomes in addition to those outlined above. The first part of the project focused on graph drawing and used a co-constructive learning approach. A small group of students worked with members of both faculties to produce a graph learning mat. This was trialled with Year 7 students, evaluated and modified before re-evaluating. The latest version will be available for use by the end of this year. This approach has the advantage of being student rather than teacher driven.
We have also used maths' lesson plan format in science. A series of Year 7 maths lessons entitled 'All About Us' introducing data handling using data collected from their class is followed up in science with 'More About Us'. Also with Year 7 we have set a joint homework involving analysis of data in the science Space topic that is immediately followed by a maths homework using the same format. This approach uses the same layout so students can visually make the links between the two subjects. Through collaboration, science teachers are able to reinforce mathematics in the context of science lessons. The maths faculty in turn are using equations from science to deliver real life algebra.
Without doubt, the most fundamental change has been in the language of maths in science. This came with the recognition that science terminology had been contradicting what was taught in maths and vice versa. For instance, what is the difference between a line and a scatter graph? Can a line of best fit be curved? When does a bar chart become a histogram?
Two years into this work, we feel that we have really only just started on this initiative and are planning for the future. We have produced a numeracy course for GCSE students that will be focused on the lower ability groups. These lessons will be taught throughout the year either as standalone lessons or where they fit into the schemes of work. The benefit of each approach will be assessed next year. Maths are planning a whole-school activity 'bringing equations to life', which we will collaborate on. We will of course continue to build numeracy goals into our schemes of work and take every opportunity to highlight the fundamental links between the subjects and to encourage and support students to apply their knowledge effectively across the broader curriculum.

## Appendix: Joint science and mathematics home learning document

## Science: Mass and Weight

## Information

- Force ( N ) = Weight ( N )
- Weight $(\mathrm{N})=$ mass $(\mathrm{kg}) \times$ gravitational field strength ( $\mathrm{N} / \mathrm{kg}$ )

Gravity is a natural force that attracts objects to each other.
Two factors determine the size of the gravitational force between two objects:

1) Their masses
2) The separation distance between them.

Gravity is the pull toward the centre of an object; let's say, of a planet or a moon. When you weigh yourself, you are measuring the amount of gravitational attraction on you by Earth. The Moon has a weaker gravitational attraction than Earth. So, you should weigh less on the Moon.


My WEIGHT on Earth is around 560 N


My WEIGHT on the moon is around 90 N

| Mass (kg) of <br> textbook | Planet | Gravitational Field <br> Strength (N/kg) | Radius of <br> planet (km) | Weight (N) |
| :--- | :--- | :--- | :--- | :--- |
|  | Mercury | 3.8 | 2,440 |  |
|  | Venus | 8.8 | 6,052 |  |
|  | Earth | 9.8 | 6,371 |  |
|  | Moon | 1.6 | 1,737 |  |
|  | Mars | 3.8 | 3,390 |  |
|  | Jupiter | 25 | 69,911 |  |
|  | Saturn | 10.4 | 58,232 |  |
|  | Uranus | 10.4 | 25,362 |  |
|  | Neptune | 13.8 | 24,622 |  |
|  | Pluto | 1.6 | 1180 |  |

## Questions



1. The mass of a textbook on Earth is 300 g .use the equation at the top in the page to show what its weight on different planets will be. Show your working when filling in the table above.
2. Identify a planet that has a similar gravitational attraction as Earth.
3. List the planets in order of gravitational force from smallest to biggest
4. Another student claims that the moon's gravity is $1 / 6$ of the Earth's gravity. Is this a true statement? Look at the chart and use mathematics to support your answer.
5. Draw a bar graph of planet against Gravitational field strength.
6. Draw a line graph of the gravitational field strength of the planets and the radius of the planets.
7. What conclusions can you make about the gravitational field strength of the planets and the radius of the planets?

## Mathematics: Speed, Distance and Time

## Information

- $\quad$ Speed $=$ Distance/Time
- $\quad$ Distance $=$ Speed $\times$ Time
- Time $=$ Distance/Speed (Delete as appropriate!)

Speed is...
The faster you travel the less time a journey will take.


The current time line is a mission to Mars in 2030
http://www.distancetomars.com/
Between 33.9 million miles and 249 million miles apart.
In 2018 they will be 35.8 million miles apart.

You are going to be planning a journey based on the average distance of 140 million miles. Work out the missing values.

| Mode of <br> transport | Speed | Time taken in <br> hours | Time taken <br> in days | Time taken in years, months, days, <br> hours, minutes and seconds. |
| :--- | :--- | :--- | :--- | :--- |
| Walking | 4.5 mph |  |  |  |
| Car at <br> maximum <br> UK speed <br> limit |  | 2000000 Hours |  |  |
| Current <br> fastest road <br> car | 270 mph |  |  |  |
| Commercial <br> Jet | 580 mph |  |  |  |
| Typhoon <br> fighter Jet | 90322.58 hours |  |  |  |
| Fastest <br> manmade <br> spacecraft | $36,000 \mathrm{mph}$ |  |  |  |
| Speed of <br> light | 186,282 <br> miles per <br> second |  |  |  |

## Questions



1. What is the closest distance between Mars and the Earth?
2. What is the furthest distance between Mars and the Earth?
3. How long would it take for the fastest manmade spacecraft to get to Jupiter?
4. Draw a distance time graph to represent the return journey to and from Mars

## Answers

| Mode of <br> transport | Speed | Time taken in <br> hours | Time taken in <br> days | Time taken in years, months, days, hours, <br> minutes and seconds. |
| :--- | :--- | :--- | :--- | :--- |
| Walking | 4.5 mph | 31111111.11 hrs | 1296296.296 <br> days | 3551 Years 181 days 7 hours 6 minutes and <br> 14.4 seconds |
| Car <br> traveling at <br> speed | 70 mph | 2000000 |  |  |
| Current <br> fastest road <br> car | 270 mph | 518518.5 hours |  |  |
| Commercial <br> Jet | 580 mph | 241379.31 hours |  |  |
| Typhoon <br> fighter Jet | 1550 mph | 90322.58 hours |  |  |
| Fastest <br> manmade <br> spacecraft | $36,000 \mathrm{mph}$ | 90322.58 hours |  |  |
| Speed of <br> light | 186,282 <br> miles per <br> second | 0.2 hours <br> 751.55 seconds |  |  |

## Appendix: Example of a student graph learning mat



# 2. Deriving quantities from gradients 

This is an account written directly by a teacher; a commentary by the editor is provided on page $\underline{6}$.

## Background and context

The school is an academically selective, independent co-educational day school in a university town. All physics lessons are taught by physics specialists, with a strong emphasis on practical work. In 2014, of 152 candidates sitting the CIE Physics IGCSE, 109 were awarded an A* grade and a further 27 an A grade. Students begin studying the IGCSE syllabus in Year 9.
Three mixed-ability Year 9 physics classes each of 22 or 23 students were involved in this study. The students had a combined mean MidYIS score of 117 with a standard deviation of 12 . For reference, the independent school average MidYIS score is 100.

## Focus of the case study

The case study focused on developing students' understanding of the concept of direct proportionality and their ability to calculate the gradient of a straight line graph and relate it to a physical quantity. The case study took place as part of a series of lessons on Hooke's Law and stretching materials. Students carried out an experiment to measure the extension of a spring as a function of increasing load and were then asked to plot a graph to show the relationship between the two variables, draw an appropriate best-fit line and identify the limit of proportionality, then calculate the gradient of the graph and constant of the spring (Appendix 1). Although students had already met the concept of direct proportionality and the link to three variable equations earlier in the Year 9 scheme of work in the context of Snell's law and Ohm's law, calculating the gradient of a graph was left at that stage as an extension exercise for the more mathematically able students in the class. Students had all previously covered the calculation of gradients and the equation of a straight line in Year 9 mathematics lessons.
A simple questionnaire to assess students' relevant mathematical skills was distributed at the start of the first lesson on the topic (Appendix 2). The results of the questionnaire (Tables 1 and 2 of Appendix 3) identified that a major difficulty for students is being able to articulate mathematical concepts and link them to real world situations rather than merely applying methods to standard problems. $67 \%$ of the 68 students in the study were able to calculate the gradient of a graph given 2 points on the line and $75 \%$ of students were able to calculate the gradient of a straight line graph, although only $44 \%$ of students gave a description of how a gradient could be determined in response to the question 'what is meant by the gradient of a line?' rather than just referring to the concept of steepness. Only 9 students were able to correctly describe what is meant if two variables are directly proportional, with a further 30 students going some way to describing the concept but giving answers that lacked precision. The better answers referred to the idea of a straight line graph passing through
the origin or a constant ratio between the two variables, while answers marked as partially correct were more vague, for example 'if the first variable increases the second variable increases' and 'they have a direct effect on each other'. A similar questionnaire at the end of the topic (Appendix 3) sought to investigate whether the activity had impacted on students' understanding.
In assessing students' work throughout the lesson and in marking the analysis completed for homework, it became apparent that there were two other areas of major difficulty faced by many students:

- The concept that a gradient could (and indeed should in this particular case) have units
- Relating the gradient of the graph to the spring constant in the equation $\mathrm{F}=\mathrm{kx}$


## Intervention

A lesson on Hooke's Law was taught to all three Year 9 classes, following the same format except for variation in whether students were asked to draw their graph and calculate the gradient by hand or using Excel. The rationale was to determine whether by allowing students to use the best-fit line function in Excel, and hence generate a value for the gradient without having to calculate it by hand, the concept would be made more accessible to the less mathematically able students in the class. One class was asked to only plot the graph by hand, another class produced the graph using Excel only while the third class was asked to use both methods of analysis. It should be noted that Excel had not otherwise been used in physics lessons during the year and on explanation of the task many students expressed concern that they did not know how to use Excel. A decision was therefore made to give students a brief demonstration in class of the skills needed to complete the analysis, with further instructions (Appendix 5) posted on the virtual learning environment for students to access for assistance with their homework if required.
Table 4 of Appendix 3 summarises the students' relative successes in their analysis of the experimental data. While most students ( $86 \%$ ) were able to draw an appropriate best fit line on the graph and describe the relationship between the variables shown by the graph ( $75 \%$ using the term 'directly proportional' and a further $17 \%$ without using that specific term), only $61 \%$ of those attempting the question were able to calculate the spring constant from the data and only $41 \%$ gave the spring constant the appropriate units. The results broadly followed the students' mathematical ability as shown by their set for mathematics and results in the internal Year 9 exam. It is worth noting also that around half of students who successfully calculated the spring constant did it using the equation F $=\mathrm{kx}$ and only one pair of data points, rather than using the gradient to take an average over a larger range of data. There was no significant difference between students' success in calculating the spring constant in the different classes.
A possible reason for the lack of the anticipated difference in results due to the use of Excel is that, of 22 students in the class that used Excel to plot the graph, 14 students still persisted in calculating the gradient manually rather than using the trend line function in Excel as they had been instructed to do. Similarly, in the class plotting the graph in Excel and by hand, 14 of 23 students chose to calculate the gradient manually. Table 5 of Appendix 3 reports student preferences for graphing by hand or using Excel from questionnaire 2, with more students who did use Excel appreciating its potential. The most common reasons given for a preference for Excel were that it is faster and (according to the students) more accurate. Those who preferred drawing the graph by hand sometimes claimed that it was quicker, stated that it was easier, that they did not like Excel or, from some of the weaker students, that it meant they understood what was going on. The results suggest that a lack of familiarity with the use of Excel prevented its use as an effective learning tool to assist with the concepts of proportionality and gradient for many of the students, and hence that if analysis using a spreadsheet
is going to be used successfully to support less mathematically able students then its use needs to be more frequent, and students need to be supported in using it in an environment where they are able to ask for help when required.
In the questionnaire distributed at the end of the series of lessons (Appendix 4), results (Table 4 of Appendix 3) showed that more students, but still not all, were able to explain the concepts of gradient and direct proportionality as they relate to physics after the sequence of lessons. After marking the students' work, an additional related activity was used as a starter exercise for the next lesson with all classes, reinforcing the key skill of calculating a gradient and assigning appropriate units. Students were shown a slide (Appendix 6) relating to an unfamiliar Physical situation and asked to calculate the gradient. Over $80 \%$ of the students were able to accomplish this given 2 minutes' think time, and perhaps incorporating similar short mathematical starter activities into lessons more frequently is a way to encourage students to make links between their knowledge of mathematics and what they are learning in physics.

## Conclusion

The study has shown that many Year 9 students struggle to relate what they have learnt about calculating gradients in mathematics lessons to real physical situations and to articulate their understanding of mathematical concepts such as gradient and proportionality. Most students were also initially uncomfortable with the idea of a gradient having units rather than being a ratio, perhaps highlighting the importance of real world examples in mathematics lessons. The wide variation in students' mathematical ability and their understanding of how to apply concepts to physics also suggests a need to consider differentiation in the physics classroom extremely carefully, particularly when mathematical concepts are being employed, and to effectively assess the prior learning of students at all ends of the ability range. A straightforward change to the lessons as taught to hopefully make the mathematics more accessible would be to add additional scaffolding to the experiment worksheet to guide students through the calculation of the spring constant from their graph for the first experiment, perhaps then asking students to repeat the experiment for springs in series or parallel without the same level of detail.
As the relationship between gradients and physical quantities poses considerable challenges for many students, every opportunity should also be taken to allow all students to practice the skill in physics lessons and to explicitly draw links with equations. Excel is perhaps best used as a complementary tool to drawing graphs by hand as, while it may save some effort with the mathematics calculations involved, an understanding of what a gradient actually is can be better achieved when calculating it by hand and students do, in any case, need to be able to calculate gradients for exams. If Excel is to be used at all, regular use is needed to give students the confidence arising from familiarity.

## Appendix 1: Experiment worksheet

## Experiment: Stretching a spring

In this experiment you will stretch a spring by hanging masses from one end. The force on the spring is equal to the weight of the masses. This force, sometimes called the load, can be calculated from the total mass hanging on the spring:

## Force on spring $=$ mass on spring (in $\mathbf{k g}$ ) $\mathbf{x g}$

Take $\mathbf{g}$ to be $\mathbf{1 0} \mathbf{N} / \mathbf{k g}$ at the surface of the earth. Force is in Newtons and mass is in $\mathbf{k g}$. As the force increases the spring stretches more. The extension of the spring is the difference between its stretched and un-stretched lengths.

Extension = stretched length $\boldsymbol{-}$ unstretched length

## Method:

1. Put on your safety glasses.
2. Use the metre rule to accurately measure the length of the spring with no weight on it. This gives the un-stretched length, $\mathrm{I}_{0}$, which you should record below, in cm :

Unstretched length of spring, $I_{0}=$ $\qquad$

Draw a diagram to show how you measured $\mathrm{I}_{0}$.
3. Hang the mass hanger on the spring. This has a mass of 100 g . Work out the force on the spring due to this mass, and write this in the appropriate place in the table of results overleaf.
4. Measure the length of the stretched spring. From this you can work out the extension. Write these in columns 3 and 4 of the table.
5. Complete the column headings in the table.
6. Add 100 g masses to the hanger, one at a time up to a maximum of 800 g , and measure the length of the stretched spring for each different mass.
7. Record your results in the table overleaf, calculating the force and extension for each different mass that was hung from the spring.
8. DON’T PACK UP YOUR EQUIPMENT YET!

Table of Results:

| Mass on spring <br> $\mathbf{m} /$ | Force on spring <br> F / | Stretched length <br> I/ | Extension of spring <br> x/ |
| :---: | :---: | :---: | :---: |
| $\mathbf{1 0 0}$ |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |

## You should all....

1. On graph paper or using Excel, plot a graph of Force against Extension. Label the axes and units and draw a line of best fit for the points. If you use Excel, print the graph off and draw the line of best fit by hand.
2. Describe the relationship between Force and Extension for your spring. Justify your answer by reference to your graph.
3. Calculate the spring constant for your spring and label the limit of proportionality (if it has been reached)
4. Describe how you ensured that your measurements were accurate.

## Most of you will also...

1. Using your graph (or otherwise) make predictions of lengths and extensions for values in the table below
2. Measure actual lengths and calculate actual extensions.

| Mass on spring <br> $\mathrm{m} /$ | Force on spring <br> $\mathrm{F} /$ | Predicted <br> stretched <br> length, I/ | Predicted <br> extension of <br> spring, $\mathrm{x} /$ | Measured <br> stretched <br> length, I/ | Measured <br> extension of <br> spring, $\mathrm{x} /$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 900 |  |  |  |  |  |
| 1000 |  |  |  |  |  |
| 1100 |  |  |  |  |  |
| 1200 |  |  |  |  |  |

3. Does your spring follow your predictions? Justify your answer with reference to your results. Make sure that the limit of proportionality is marked on your graph.

## Some of you will also...

Investigate springs connected in series and parallel. Consider how best to both qualitatively and quantitatively compare your results with those for one spring. and/or
Repeat the experiment above but with copper wire or rubber bands rather than a spring.

## Appendix 2: Start of project questionnaire

## Part 1 - About You

Name:
Form:
Maths set:
Mark in Y9 Maths exam:
Mark in Y9 Physics exam:

## Part 2 - Some questions

1. Rearrange the equation $x=y z$ to make $y$ the subject
2. Rearrange the equation $F=k x$ to make $x$ the subject
3. Rearrange the equation $I=V / R$ to make $R$ the subject
4. What is meant by the gradient of a line?
5. Calculate the gradient of a line which passes through $(4,2)$ and $(8,4)$
6. Calculate the gradient of the graph below

7. What does it mean if two variables are directly proportional?
8. If $F$ is directly proportional to $a$ with a constant of proportionality $m$, write an equation linking $F, m$ and $a$.

## Appendix 3: Selected summary results from questionnaire and student work

Table 1: summary of responses to questionnaire 1

| Question | Number of students giving correct answer | Number of students giving partially correct answer | Number of students giving incorrect answer or not attempting question |
| :---: | :---: | :---: | :---: |
| Rearrange the equation $x=y z$ to make $y$ the subject | 64 | 0 | 4 |
| Rearrange the equation $F=k x$ to make $x$ the subject | 63 | 0 | 5 |
| Rearrange the equation $I=V / R$ to make $R$ the subject | 55 | 0 | 13 |
| What is meant by the gradient of a line? | 30 | 36 | 2 |
| Calculate the gradient of a line which passes through $(4,2)$ and $(8,4)$ | 46 | 2 | 20 |
| Calculate the gradient of the graph | 51 | 1 | 16 |
| What does it mean if two variables are directly proportional? | 9 | 30 | 29 |
| If $F$ is directly proportional to $a$ with a constant of proportionality $m$, write an equation linking $F, m$ and $a$ | 34 | 8 | 26 |

Table 2: student responses to question 'what is meant by the gradient of a line?'

| Criterion | Number of responses |
| :--- | :---: |
| Refer only to idea of steepness of line | 32 |
| Describe mathematically as change in $\mathrm{y} /$ <br> change in x | 30 |
| Relate gradient of graph to $\mathrm{y}=\mathrm{mx}+\mathrm{c}$ | 4 |
| Completely incorrect response | 2 |

Table 3: summary of student graph and analysis

| Criterion <br> (56 submissions) | Number of <br> students giving <br> correct answer | Number of <br> students giving <br> partially correct <br> answer | Number of <br> students giving <br> incorrect <br> answer or not <br> attempting <br> question |
| :--- | :---: | :---: | :---: |
| Draws appropriate best fit line on <br> graph | 48 | 5 | 3 |
| Able to describe relationship shown <br> by graph (partially correct if relationship <br> correctly described without use of the term <br> 'directly proportional') | 42 | 10 | 4 |
| Calculate spring constant from data <br> (partially correct if follow correct method but <br> either calculate gradient from x/y not y/x fail to <br> do $1 /$ gradient | 31 | 11 |  |
| Give spring conert to spring constant) <br> units |  | 03 | 9 |

Note that of 22 students in the class which used Excel to plot the graph, 14 members of the class still calculated the gradient manually rather than using the trendline function in Excel. 14 of 23 students in the class which plotted the graph by Excel and by Hand also calculated the gradient manually and not using Excel.

Table 4: changes in description of gradient and direct proportionality between questionnaires 1 and 2

| Question | Number of <br> students giving <br> correct answer |  | Number of <br> students giving <br> partially correct <br> answer in <br> questionnaire 1 |  | Number of <br> students giving <br> incorrect <br> answer or not <br> attempting <br> question |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Q1 | Q2 | $+/-$ | Q1 | Q2 | +/- | Q1 | Q2 | +/- |
| What is meant by the gradient of <br> a line? | 30 | 38 | +8 | 36 | 18 | -18 | 2 | 3 | +1 |
| What does it mean if two <br> variables are directly <br> proportional? | 9 | 19 | +10 | 30 | 22 | -8 | 29 | 18 | -9 |

Table 5: students' preferences to draw graph and calculate gradient by hand or using Excel

| Preference | Hand <br> only | Excel <br> only | Both | Total |  |
| :---: | :--- | :---: | :---: | :---: | :---: |
| Graph | Hand | 16 | 8 | 4 | 28 |
|  | Excel | 16 | 13 | 13 | 42 |
|  | No <br> preference | 0 | 1 | 1 | 2 |
|  | Hand | 14 | 9 | 7 | 30 |
|  | Excel | 7 | 12 | 12 | 31 |
|  | No <br> preference | 1 | 1 | 1 | 3 |

## Appendix 4: End of project questionnaire

## Name:

Form:

1. What is meant by the gradient of a line?
2. What does it mean if two variables are directly proportional?
3. When you collect experimental data (as for the Hooke's Law experiment) do you prefer to draw your graph by hand or using Excel? Why?
4. If you have a graph which shows that two variables are directly proportional, do you prefer to calculate the gradient by hand or using Excel? Why?

## Appendix 5: Instructions for Excel

## Drawing graphs in Excel

A simple guide

## 1. Enter your data

Just type your data in to the Excel spreadsheet
Don't forget to include column headings giving the variable name and units


## 2. Formatting your data

You can format your data using the Icons on the 'home' tab as in Microsoft word It won't affect your graph but is good practice to also adjust the number of dp your data is given to - this is especially important if you are then going to print the results table


Highlight the data, right click and then select 'format cells'

## 2. Formatting your data



This should open a dialogue box, and the 'number' tab will let you edit the number of dp numbers are given to
You may llke to experiment with some of the otheroptions too

## 3. Inserting a graph

Select your data and go to 'Insert', 'Scatter' and select the scatter graph without lines linking the points

## 4. Selecting the data in your graph

Once your graph has appeared you can right click on it and choose 'select data' to edit which data is on the $x$ and $y$ axes or to include / exclude data series


## 4. Selecting the data in your graph

Select a series (set of data) and click 'edit'


## 5. Formatting your graph

Your graph should have:

- Major and minor gridlines
- Small markers, preferably crosses
- A title
- Axes labels (Including the units of the quantity where relevant)
With the chart selected you should have the option of the 'Layout' tab at the top which will give you options for most of these




## 6. Adding a trend line

Unless your points lie on a straight line, you should print your graph and add a best-fit line by hand as Excel is not good at drawing a smooth line and tends to just joln the dots
However, assuming points lie on a straight line Excel can be very usefully used to find the gradient of the trend line
You will need to make sure before adding the trend line that any points beyond the limit of proportionality (I.e. that any points beyond the timit of proportionality (l.e. consistently lying off the straight line at
It may be easiest to do this by creating another graph with
just the data that lles on a stralght line selected (see 4
Selecting the data in your graph), or just create a new
graph by only entering the data that lies close to the
stralght line in a new sheet

## 6. Adding a trend line

A trend line (line of best fit) will now appear on your graph. Check it looks sensible! Double click on the trend line to open the Format Trendline box and select 'Display Equation on chart' at the bottom to show the equation on the chart


## 7. Working out the gradient

Don't forget that a gradient has units, which will be the units of the variable on the $y$-axis divided by the variable on the $x$-axis (so here $\mathrm{N} / \mathrm{cm}$ )


## 7. Working out the gradient

Excel will then display the equation of the line of best fit on the chart in the form $y=m x+c$ The gradient is therefore the number appearing in front of $x$, so in this case is 0.4781


## 8. Finding the spring constant

We will discuss more in our next lesson, but as $F=k x, k$ (the spring constant) $=F / x$ So if you have plotted $F$ on the $y$-axis and extension ( $x$ ) on the $x$-axis as shown below the gradient gives you the spring constant If you have plotted $F$ on the $x$-axis and extension on the $y$-axis then the gradient is $1 /$ spring constant, so you should do $1 /$ gradient to find the spring constant Be careful of units!

## Appendix 6: Starter question to find the gradient

## What is the gradient?



Plate separation / m

# 3. Using a literacy approach to interpreting graphs 

This is an account written directly by a teacher; a commentary by the editor is provided on page 10.

## Introduction and background

A lack of literacy and numeracy in students is a problem encountered by the majority of science teachers I meet, especially those teaching in schools in deprived areas. Much of the focus of senior leadership teams tends to be on improving the former, which has been shown to affect students' performance across the curriculum, and I believe the latter is often overlooked. Whereas the archaic and facile interpretation of literacy as simply reading a book has begun to be replaced by a deeper sense of decoding and conveying complex ideas using language, many teachers still seem to perceive numeracy as a mechanical ritual of mathematical processes: simple arithmetic, putting numbers into formulae, plotting graphs. It is my belief that genuine numeracy, in this case the skill of interpreting graphs, leads students to a more critical understanding of the world around them, making it comparable, rather than mutually exclusive, to literacy. For this reason, I took an approach to numeracy that incorporated literacy from the beginning.

## Context and focus of the case study

I am a science teacher, specialising in physics, at an academy situated in one of the most economically deprived areas of the country, in which more than two thirds of students are eligible for free school meals. Teaching a Year 7 group with particularly weak literacy and numeracy this year, half of whom left primary school with a level 3 b in science and in which more than half of students have some form of SEN provision, I spent a considerable amount of time attempting to build their skills in both areas. This case study refers to the work I have done with the group around interpreting line graphs, a numerical skill that is crucial in all three of the major strands of science, especially with the new curriculum being introduced in September 2015.
To many in the class, a graph was little more than a series of abstract symbols and shapes when they started at the academy, much as letters and words make little sense to someone unable to read or write. The work described here was inspired by Brazilian educator-philosopher Paulo Freire's work with illiterate adults in Brazil in the 1970s; he argued that: 'Acquiring literacy does not involve memorising sentences, words or syllables ... rather an attitude of creation and re-creation' (Freire, 2013). Rather than drill a prescribed method of interpreting graphs into the students, I began by asking them to decode graphs and break them down into their constituent 'parts', before using these parts to create their own line graphs and deepen their understanding of what messages they can convey. The parts of graphs I felt were crucial in allowing students to interpret them were the axis titles, the direction of the line and the gradient; by relating each part to a linguistic device I hoped to equip them with the skills to then begin to look beyond the symbols and shapes and grasp the meaning that lay behind them.

## Strategies and outcomes

The first challenge was helping students to appreciate the axes as a link between two 'things', moving from an abstract x and y to a concrete cause and effect. The first stage in this process, which I have been doing throughout the year, is to show students graphs and ask them to describe them using socalled 'scientific sentences', which have the structure: 'The higher the ... , the higher / lower the ...'. By regularly relating graphs back to this framework, the link between literacy and numeracy became embedded in the students' brains; the graph and the sentence were simply two ways of conveying the same idea.

Once a link between line graphs and scientific sentences had been established, it was important to ensure that students understood how the direction of the line related to the concepts of positive and negative correlation. At this stage, given the low levels of literacy in the group, I felt that the nomenclature was less important than the underlying concepts, so used the terms 'more / more' and 'more / less' graphs to identify positive and negative correlation respectively. These two terms were a simple way of describing the two types of scientific sentence that the students were used to writing: 'The more (or higher) the $\ldots$, the more / less (or higher / lower) the ...'. Figure 1 shows an example of an activity that reinforced the link between the graphical and linguistic representation of the same ideas.

Figure 1

## Choose from higher / lower / stays the same



It can be seen in this activity that some students still struggled to move to a cause and effect view of graphs, writing 'the higher the rate of erosion, the higher the average wind speed' (this was a topic they'd been studying in Geography). One framework for ensuring they put the independent variable first is by pointing out that x comes before y in the alphabet, so the x variable comes before the y
variable in the scientific sentence. While this admittedly does little to improve their understanding of cause and effect, it offers a simple memory aid to help them correctly translate between graphs and language.
The final part to decode was the physical concept of rate and its relation to the graphical concept of gradient. A common problem I have observed amongst students, even top end GCSE candidates sitting higher papers, is their inability to adequately describe graphs of varying gradient; many seem to be limited to a more / more or more / less understanding, ignoring the physical implications of the changing gradient. To illustrate this, the students used model cars and ramps to try and relate the steepness of the ramp to the speed of the car; the assumption being that speed would be one of the simplest rates for them to visualise. Although many struggled to generalise this principle in later work, there was some evidence that students were beginning to appreciate the temporal implications of gradient: Figure 2 shows one student explaining why a graph they had sketched was steep by using the word 'suddenly'.

Figure 2

## Draw your own graph and tell your own story



What happened?
I was walking down the road really happy' then something heuppend and 7
was really sad


Having broken graphs down into their constituent parts, the next task was for the students to build their own graphs from those parts. I used the conceit of telling stories using graphs as a means to give them a framework around which to create their own graph. This began with modelling: borrowing an idea from another teacher at the academy, I read a story to them based on Little Red Riding Hood and the students drew a graph of how her happiness progressed over time. We repeated this with other stories, which led to the students writing their own story and sketching a graph to go with it. This was relatively successful: as can be seen in Figures 2 and 3, there was clear evidence that students understood the concepts of axes as a link between variables, how the direction of the line shows correlation and the relationship between gradient and rate.

Figure 3


Draw your own graph and tell your own story



One problem that I had not foreseen in using the story model to interpret graphs was an over-reliance on the scaffolds: the majority of the students, when asked to draw a graph of their story, used the same variables as the Little Red Riding Hood example, happiness vs time. This dependency would likely be reduced by continued practice at graph sketching with the scaffolds slowly being removed, which highlights an important point. Graphical numeracy is not a skill that is learned over the course of an hour or two; these activities must be embedded into as many lessons as possible throughout the curriculum.

## Conclusions and next steps

As outlined in the introduction, the ultimate aim of my work around numeracy at the academy has been to encourage students to critically engage with graphs, moving from simply describing them to inferring the more complex ideas they convey. Much of the work described above, with students coming from low starting points, was limited to the kind of mechanical method - translating graphical information into 'scientific sentences' - that I had wanted to avoid in the beginning, yet I felt such methods and frameworks were necessary in order to equip the students with enough skills to begin to think at a higher order. The examples of work shown above include questioning, both verbal and in written feedback, that nudged students towards a deeper exploration of the 'Why?'s behind each graph.
Other ways of moving from the simple skills of interpreting graphs to the more complex ones of analysis and explanation include linking them to practical work. Figure 4 is another example of work from the Y 7 class discussed above; despite having a poor grasp of scale, the student has nonetheless attempted to write a conclusion from the graph they have drawn, in the form of a scientific sentence. This could be developed with further questioning around why this relationship has been observed.

Figure 4


Graphs can also be related to formulae, which offers a useful way of exploring the ideas of correlation, causality and proportionality. Figure 5 shows an example, from the same Y7 class, of a student attempting to relate variables and constants in a formula to a graph and scientific sentence.

Figure 5


One final, more contentious, method of developing these ideas further is suggested by Paulo Freire: introducing politics to the classroom. Discussing graphs that inspire debate can provoke a healthy dose of intellectual doubt in students; John Taylor, founder of the EPQ qualification at A Level and an advocate of taking a 'philosophical approach to teaching', highlights the importance of 'exploring a topic about which there is real uncertainty ... not just a rhetorical ploy' (Taylor, 2012).
To give an example of this idea in practice, during a lesson on numeracy skills with a relatively low ability Y9 group, beset by numerous behaviour and attendance issues, I showed them a graph of exclusion rates for different ethnic groups. Five of the 20 students were of Caribbean heritage, including one of mixed race; asking the question why those ethnic groups had the highest exclusion rates provoked a genuine debate amongst many students who tend to have little engagement with school life. Moments like that point to the real value of numeracy in the curriculum; to use a scientific sentence, the sooner students understand that reading graphs is a fundamental part of understanding the world around them, the better.

## Bibliography

Freire, P. (2013); Education for Critical Consciousness (Bloomsbury Academic, London)
Taylor, J. L. (2012); Think Again: A Philosophical Approach to Teaching (Continuum, London)

# 4. Introducing terms used to describe data types 

This is an account written directly by a teacher; a commentary by the editor is provided on page 13.

## Background context

- Role within institution: PGCE Intern
- Key features of student profile: The school has around 1400 students, just over $50 \%$ of which are girls. Of the student population, $35 \%$ are supported by free school meals and $11.2 \%$ are supported by school action plus.
- Class context: The case study involved a mixed science class of 28 Year 7 students. They are in a top band, with levels ranging from 4 a to 6 b. There are 3 students with SEN status who needed to be taken into account.


## Focus of the case study

The focus of the case study was to develop students' ability to collect data, with the emphasis on using correct key words to describe the data that they had collected. Determining the efficiencies of different fuels by recording temperature change of water in a boiling tube provided the scientific context. The relevance of understanding how to correctly use the thermometer and the scale was explicitly obvious to the students, but the relevance of using the key words that only top set students have encountered in maths lessons was not. Students need to be aware of the type of data that they are using in order to know how best to present it. Many students struggle with choosing the best way to collect and interpret data and so challenging this head on was worthwhile. Building on this, when students start to use statistics at A Level, understanding data will be extremely important and ensuring they have collected data correctly will really benefit. The challenge for these students was the lack of exposure to these words that most students will have had, but also how to extend higher ability students who had been aware of these words from maths lessons.

## Intervention

In order to ensure that the students fully understood the meanings of the mathematical concepts that we wanted them to grasp, they were embedded within the lesson as part of an investigation. The lesson started with students reading the practical instructions and answering questions on what the dependent, independent and control variables were. This assessed prior learning and ensured that these three key words were covered. The next 10 minutes of the lesson involved a teacher led explanation of definitions of words: quantitative, qualitative, continuous and discontinuous.

Following each explanation, students were asked to show their understanding using examples. During this time, students' progress and understanding was monitored by moving throughout the classroom and supporting if necessary. An extension task was also given. The practical was demonstrated and then students carried out the work in pairs. Finally, students were required to complete the A3 sheet that asked them to interpret their data. A list of key words and definitions was provided in order that students could be supported throughout. However, there was a question on the sheet for which no scaffolding was provided to stretch more able students. This sheet was used as continual assessment during the class and sheets were marked afterwards. Fourteen students completed everything on the sheet correctly, including the extension. Of the remaining $50 \%, 9$ either did not complete the question that had no scaffolding provided or got this wrong, three answered the dependent variable incorrectly and 2 answered the staffroom hair question incorrectly. This question was designed to check that students could further apply their understanding. A few weeks later, students did a different investigation and their choice of graph depended on whether the data was continuous or discontinuous. Students voted for the choice of graph with hands up and only 4 got this incorrect, showing that they had taken some information in! Giving the students data that they had collected to analyse meant that they valued it and enjoyed it much more - particularly as the practical involved burners! It also gave them a reason to apply knowledge rather than more examples and they could explicitly see where this knowledge was valuable. If I were to redo this activity, I would stretch the investigation over 2 lessons. Some of the lower ability students struggled with the volume of new information so giving them more time to recap some words and go over others in more depth would be beneficial. Ideally, using key words throughout a topic with correct definitions then ending the topic with an investigation similar to this one would have been effective. This would check long term retention and students would be better able to apply knowledge in many different situations.

## Conclusion

As somebody who throughout school had very little confidence in my own mathematic ability and was scared by the use of it within science lessons, I knew not to just throw it at students and expect them to follow. The top set mathematicians understood quickly and moved onto the extension tasks, challenging themselves further. But scaffolding meant the rest of the class were able to move at a slower pace with me to guide them and their confidence and skills improved as a result. Whilst I am aware this is not ground breaking, I think having that lack of confidence really emphasised the need for scaffolding. It really struck me during the experience how little the two departments talk and so just a few links embedded in lessons and an awareness of subject overlap could be a really powerful tool. Having a conversation with the maths department ensured that I was aware that some students would already know these concepts and could plan accordingly. I will continue thinking about incorporating maths skills within my lessons. Little and often seems to be the best way so as not to scare the less confident students but to ensure that there is that link. It is important that students see that but it is equally important not to scare them away from the science we need them to do. By linking these two subjects more clearly, we can support students in both subjects.

## Appendix: Language of maths: collecting data

## Language of maths: collecting data

Different ways of collecting data - counting and measuring qualitative and quantitative data. Measuring instruments and scales. Collecting sets of data - variables - different types of data.

Key words: quantity, value, unit, quantitative data, qualitative data, significant figures, variable, continuous data, discrete data, categorical data, time series, grouped data, primary data, secondary data.


I sampled how long everyone's hair was in the staffroom. Is this continuous or discrete data?

## Fuel burner practical

- There are 4 liquids that can be used as fuels: hexane, butane, propane and methanol.
- You are going to measure which is more efficient.
- Half fill a boiling tube of water, then take the temperature of the water. Record this.
- Place the fuel burner under the boiling tube and time.
- Check how much the temperature has risen and record this.


DNA: read the instructions for the practical and answer the questions

How will we tell which fuel is more efficient?
Which is your independent and dependent variable?

## First key words:

- Independent variable
- Dependent variable
- Control Variable


Helps keep it a fair test!

| Key Word | Definition |
| :--- | :--- |
|  | The thing YOU change |
|  | The thing you are measuring |
|  | What you keep the same to make it a fair <br> test |
| Discrete data | Data that can only take certain values |
| Primary data | Data you have taken yourself |
| Secondary data | Data you have taken from someone else |
| Qualitative data | Descriptions (results are words) |
| Quantitative data | Counting (results are numbers) |
| Continuous | Data that can take any value (within a <br> range) |

## Second key words:

- Quantitative data - counting (results are numbers)
- Qualitative data - descriptions (results are words)



## More key words!

- Data can be...

Discrete data:

## Continuous data:



If we measured the shoe size of everyone in this class, would that data be discrete or continuous?

What about if we measure their heights?

## 4 examples

-What distance do year 7 s travel to school?

- How many students prefer dogs to cats?
- What was the amount of rainfall in England in 2014?
- How many legs do different animals have?

Extension: think of another example yourself

## Practical

- Record your results in the table on your sheet



## More key words!

- Primary data - something that you have collected
- Secondary data - results you have gotten from someone else



# 5. Joint mathematics and science day to teach equations and graphs 

This is an account written directly by a teacher; a commentary by the editor is provided on page 17.

The purpose of this project is to raise awareness of students' transfer of skills from maths to science lessons. In maths lessons students understand the basic concept of rearranging formulae and how to draw and interpret graphs. When they are then asked to repeat the same skills in a science lesson they struggle to transfer these skills across. Two maths teachers and I were present for the entirety of the day for this collaborative venture. 60 students from the top 2 maths sets in Year 9 were chosen to participate.
The key skills that this particular project focused on were how to use scientific formulae to calculate an answer and then further calculate answers by rearranging each formula. Another major skill focus was on drawing and interpreting data from a graph.
The chosen theme was around cars and car chases, which allowed the major concepts of speed, velocity, acceleration, distance, time and kinetic energy to be introduced. These are major concepts for both maths and science GCSE curricula. The project started with students discussing what the purpose of speed limits was, this then led on to stopping distances. Students were placed into groups of 6-7 and then asked to write down factors that firstly affect thinking distance, then secondly braking distance.
From this the idea developed into the introduction of how to calculate kinetic energy using the scientific formula and students then had to complete a table of results when they were given the mass and speed of a car.


Students were then asked to draw a graph of their results and told upon which axis to put which variable as this is not taught in maths. Once they had completed their graphs, students were shown an illustration of what a successful graph should look like.



## Keywords myelocity, distance, acceleration, deceleration

## Marking Points

The following points need to be seen on your graph:

- Title
- Correctly labelled axes
- Units for the independent and dependant variable
- A scale written in units of $1,2,5$ or 10
- Data points should fill $80 \%$ of the graph space
- Accurately plotted data points (cross should be within 1 mm of the data value)
- A suitable line of best fit


Students correctly identified that they needed to draw a line of best fit and drew a line graph as they had continuous data. The majority of students correctly drew a curved line of best fit with only a couple drawing a straight line. This was further discussed with the maths teachers who explained to the students that because to calculate kinetic energy the velocity is squared, this will lead to an exponential graph plot which means it will be a curved line.

Students were next introduced to how to rearrange scientific formulae. The first formula used was calculating speed, distance and time. This is familiar to all students from Key Stage 3 and a concept used on a daily basis. The triangle method was used as this is an easy starting point to use to show how to rearrange formulae and it is a method also taught in maths. Students were then asked to manipulate the formula to be able to calculate either speed, distance or time.


Following on from this, students were introduced to distance-time graphs and how to interpret data from them. This was followed by questioning to gauge their understanding of data interpretation.




Students were next asked to use the scientific formula for calculating acceleration to calculate either acceleration, change in velocity or time taken. Students had not seen or used this formula before, but having previously been shown how to use the triangle method to rearrange a formula this was used to test their manipulation skills further.


All students were able to correctly calculate the answers using the triangle method and remembered to use the correct units at the end of their answers.

The next stage of the process was to look at velocity-time graphs and how to interpret them.
Caution was taken not to confuse students with distance-time graphs when reading them. This was emphasised to students. Key points such as a straight line on a distance-time graph meaning it is a stationary object, whereas a straight line on a velocity-time graph means an object is moving at a constant velocity.


## Acceleration $=\frac{\text { change in vaiocity }}{\text { time taten }}$



1. $2 \mathrm{~m} / \mathrm{s}^{2}$
2. $40 \mathrm{~m} / \mathrm{s}$
3. $120 \mathrm{~m} / \mathrm{s}$
4. $2 \mathrm{~m} / \mathrm{s}^{2}$
5. 

$3.3 \mathrm{~m} / \mathrm{s}^{2}$
4.
4.

The next phase of the day was an independent skills based task. Students were given 4 film clips of car chases to analyse. They were asked to use google maps to plot the route of the car chase to determine an approximate distance. They then watched the clip on YouTube to establish the overall duration of the chase. From these values they were then asked to calculate the average speed for the journey.
The 4 films were Bullit, the Bourne identity, Quantum of Solace and the Italian job. Students were given a map of where each film was based to sketch upon and a printout of directions showing the route of the chase.


Once students had calculated the average speed for each clip, they were then asked to make a presentation to explain how they had calculated their answer. This included using the correct formula and then explaining how realistic and reliable each film car chase was compared to a real life situation. This allowed students to consolidate what they had learnt over the course of the day and apply it to an everyday situation.

## EQUATIONS:

Speed $=$ distance $\div$ time
Distance $=$ speed $\boldsymbol{*}$ time
Time $=$ distance $\div$ speed


This is how we calculated the speeds the cars would have been travelling at in the car chases. We were then able to compare these speeds with the actual speeds that would have been possible, to see if they were realistic.

## QUANTUM OF SOLACE

This car chase is even more impossible than Bourne Identity as the car would have to travel at $2238 \mathrm{~m} / \mathrm{s}$ ( $5,006 \mathrm{mph}$ ) to complete the 535 km journey in 239 seconds. This is whilst dodging traffic and turning corners, meaning that Bond would have to travel faster than this for his average to be this high. This would have made it impossible to film and would have made it impossible for Bond to control the car.


This chase is very unlikely as the car would have had to travel at over 105 mph to complete the journey in 15 minutes and 27 seconds which involves turning corners that they would have to slow down for. This would be even more unlikely as the car that is being driven appears to be old and not in the best condition.


This film clip isn't reliable as it is impossible for a car to go the speed at which it was travelling at in this clip. It also isn't reliable because it is even more impossible than the Bourne Identity clip which is slightly impossible/almost possible in comparison to how



To conclude, I feel this was a successful collaboration. By having both maths and science teachers present this helped to emphasise to students that the skills were applicable to both subjects. By incorporating scientific formulae and then applying these to graph drawing and analysis, this helped students to consolidate skills learned through the course of the day. A further point is that ensuring that both the maths and science departments introduce new skills and methods of application in the same manner helps to ensure these skills are successfully transferred between the two subjects in a consistent manner.

Student comments from the day: 'This has been really useful as it makes the links between the two subjects very clear.'.
'I feel much more confident at being able to transfer my skills from maths lessons to science lessons.'
'Now I am able to rearrange scientific formulas easily using the triangle method.'
'This day has helped improve my knowledge of how to correctly draw and interpret graphs.'

# 6. The vocabulary of graphs - an example of departmental collaboration 

This is an account written directly by a teacher; a commentary by the editor is provided on page 20.

## Background context

The authors are lead practitioners for science and mathematics working in a mixed 11-18 comprehensive school in a small market town, taking nearly all children from the town.

## Focus of the case study

The mathematical skill identified was to look at the type of correlation produced in a set of data, particularly with reference to describing the trend or pattern in the data and using this to comment on how appropriate the range was that had been chosen. Students found it hard to identify trends if the correlation was not a simple straight line. They had problems adding lines to graphs when points were arranged in a curve, and also had problems identifying more complex relationships in data such as when 'the results go up in twos'. Students were asked to comment on the differences between the use of key terminology in maths and science, with no student correctly identifying correlation as meaning anything other than 'something to do with scatter graphs' or a 'relationship between numbers'.
Students could write a correct conclusion based on the line of best fit that they had drawn in science, with typical comments saying as the number of carbon atoms in a fuel increases, so does the amount of energy released', but they could not see the link between this and the word correlation they had used in maths.
In discussion between the two teachers involved we also identified that: (a) in maths lessons they only ever drew straight lines of best fit and looked at linear relationships, leading students to do this in science; (b) maths teachers very rarely commented on the relationships between variables and did not use the word variables often; (c) in maths ranges were only used in data and never to describe the range of values chosen for a variable.

## Context of the case study

The group chosen were a Year 9 science set 3 (out of 5), who commonly have now got GCSE target grades of between A-D. There was not an equivalent maths group containing the same students, although all students were asked for their maths experiences in the science lesson and the maths teacher involved did teach a handful of the students involved.

## Intervention

After drawing some initial 'baseline' graphs of sets of data, the two teachers involved in the projects planned several activities.

## In science

We planned to identify the meaning of key words at the start of the project by discussing what they thought at the start of the project and what they might know as a class that contradicted those initial ideas ( to all start at the same point). This allowed some students to bring out problems they had with the language and we brainstormed what issues they had with the terminology used and the problems they had in drawing graphs. We then identified the 4 most important terms they needed as being correlation, line of best fit, pattern and range. We sorted out meanings that were the best from a set of possible true or false statements, for example 'to have a pattern from a set of data, we must we able to see it goes up in even amounts e.g. in $10 s$ ' or 'a positive correlation must always be a straight line going up'. After deciding on a good definition of each of these words we then looked at some example graphs to then decide what we would have previously said and what we now think, then returning to our start graph and drawing our curve and then writing a conclusion based on the data (also commenting on the range of our independent variable). We then peer assessed the graphs and took pictures to look at showing particularly good examples of progress from the first to last graph.

## In maths

The schemes of work are to be changed to allow the following to happen in future:

1. Lines of best fit to be extended to include less obvious 'lines' with anomalous data and curves to also be included. The use of the word 'pattern' is to be brought in to the explanation of what line of best fit might be chosen.
2. Maths teachers will also refer to the labelling of the two axes as 'variables' to help back up the teaching in science.
3. We will do a joint summer Year 8 project to generate some real life data in science and then liaise with the maths teachers to draw and describe graphs based on this data, with joint planning and a small group of interested teachers taking part. This has previously run within the school, but had not involved enough planning on the use of language and often ended up being unpopular with teachers due to the lack of a very simple aim and a lack of perceived usefulness.

## Conclusion

The project had a very great impact on student understanding, with students commenting that they had always found graphs confusing due to the separate approaches the two subjects took. The most common comments included 'I thought you were talking about different things' or surprisingly 'I didn't know maths were asking us to plot graphs of actual experiments'! The graphs and use of language around the conclusion or evaluation sections were very much improved and in line with GCSE expectations for both subjects, with students using the word correlation correctly and talking about the confidence they had in the conclusion. In future, we will start the joint project and both do department meetings to allow both sets of staff to learn from what we have done. We will also produce key word posters to use on the wall in both subjects to refer to when teaching graphs in both subjects.

## 7. Molar calculations in chemistry

This is an account written directly by a teacher; a commentary by the editor is provided on page $\underline{23}$.

## Focus of the case study

The case study focuses on the application of proportion to the teaching of GCSE molar calculations. In particular, there is a need for students to be able to find the proportion by mass of an element within a compound and to be able to calculate the maximum mass (assuming $100 \%$ yield) of a product that can be made from a given mass of reactant or the mass of a reactant required to make a given mass of product (reacting masses calculations).

The underlying chemical concept is the understanding that a chemical equation simply states the proportions (by numbers of atoms, molecules, formula units or ions) in which materials react.
My experience has shown that even able students can struggle to understand the significance of a chemical equation and how to use it to calculate chemically significant quantities. This results in poor retention of calculation skills and a disproportionate amount of time being devoted to these fairly mechanical skills.

## Context of the case study

In this school, students sit the GCSE at the end of Year 10. Chemical calculations are first taught at the end of Year 9 or the beginning of Year 10. The cohort of able students is streamed into upper and lower sets based on outcomes in Year 8. Class sizes are approximately 30 students and the strategy to be discussed is designed for teaching to a whole class.

## Intervention

Students are taught proportion in mathematics in Year 8 so, by the time they start GCSE, they have the mathematical skills to be able to complete molar calculations. The strategy is based on the supposition that the blocks are likely to be due to:

- trying to assimilate the concept of the mole at the same time as carrying out calculations;
- not recognising that they have the skills to be able to complete the molar calculations.

Up until now, I have approached this topic by first teaching an understanding of the mole as a quantity used in chemistry. I give them Avogadro's constant, along with the concept of relative mass being based on a scale where $1 / 12^{\text {th }}$ of the mass of an atom of ${ }^{12} \mathrm{C}$ is assigned a mass of 1 unit. I then proceed to teach the calculations in order of increasing complexity:

1. Relative formula mass
2. Percentage by mass of an element in a compound
3. The actual mass of an element in a given mass of compound
4. Reacting masses

Teaching has taken a 3 step approach. For example, when carrying out a reacting mass calculation:

1. Use mass of reactant to calculate number of moles of the reactant.
2. Use the balanced chemical equation to find the number of moles of the product that can be formed.
3. Use this number of moles to calculate the mass of the product formed.

This involves the use of a formula $n$ (number of moles) $=m$ (mass of material)/ $M$ (mass of 1 mole)
Students sometimes get 'hung up' about this formula and try to remember it using a calculation triangle, leading to difficulties in stages 1 and 3 of the calculation. They can also find step 2 difficult, probably because of a lack of a firm understanding of what the numbers in a balanced chemical equation signify.
In the revised method, there is no need for a formula to be used. Detailed discussion of the concept of the mole is deferred. At the start students are simply told that the mole is the quantity chemists work with and that it is a large number of atoms/molecules/formula units or ions. The practical use of the calculations to find useful quantities is emphasised. Students are also told how to find the relative mass of an element from the periodic table and that the actual mass of a mole is the relative mass in grams. The more detailed understanding of the nature of the mole is returned to once the method for the chemical calculations is mastered.

The core of the concept is that teaching concentrates on helping students to recognise that the concepts involved in chemical calculations are exactly the same as they have been previously taught in Year 8 mathematics. This is facilitated by teaching in the same manner as employed by mathematics and by maths teachers having used chemical examples in their initial concept teaching.
In mathematics, proportion is taught using a table. Two examples are given:

1. Finding relative formula mass/mass of a mole/percentages by mass/actual mass of an element in a given mass of compound.
The table illustrates, for example, the method used to calculate the mass of carbon in 300 g of carbon dioxide.
$\mathrm{CO}_{2}$-percentage masses of elements

2. Reacting masses

The illustration demonstrates the method for calculating the maximum mass of water produced by combustion of 100 g of ethane and the maximum mass of carbon dioxide produced by combustion of 25 kg of ethane.

## The combustion of ethane



| Mass $\mathrm{C}_{2} \mathrm{H}_{6} / \mathrm{g}$ | Mass $\mathrm{O}_{2} / \mathrm{B}$ |  | Mass $\mathrm{CO}_{2} / \mathrm{B}$ | Mass $\mathrm{H}_{2} \mathrm{O} / \mathrm{B}$ |
| :--- | :--- | :--- | :--- | :--- |
| 100 | Excess | $\frac{176}{60} \times 100=293$ |  |  |
| 25000 | Excess |  |  | $\frac{108}{60} \times 25000=45000$ |

This initiative is new to this school and the impact will take 2 years to fully work through. However, initial results are promising. The method has been used to teach a Year 9 lower set. Students responded positively, and readily mastered the calculations; retention of the skill is yet to be assessed.

## Conclusion

The full impact of this initiative is yet to be demonstrated. However, initial results are promising. Next steps are to work with the mathematics and chemistry departments to ensure the methods are fully embedded. I also plan to recruit volunteers from last year's GCSE cohort, who had been taught by the previous method and who found chemical calculations difficult, to assess whether the new method is likely to improve outcomes.

## 8. Interpreting graphs

This is an account written directly by a teacher; a commentary by the editor is provided on page 26.

## Background

Students were in the summer term of Year 9 and were completing their Key Stage 3 studies following an in-house course designed to prepare them for GCSEs at Key Stage 4. All students were girls and were in the highest ability teaching group. Science lessons at the school use objectives based around Bloom's Taxonomy through the use of 'IDEALS'. This system is applied by using consistent command words at increasing levels of thinking skills as per Bloom's philosophy:

- Identify (such as naming or labelling)
- Describe (observation skills)
- Explain (with emphasis on 'because' and 'therefore' in answers using scientific ideas)
- Analyse (pros and cons etc.)
- Link (taking ideas from other topics or wider concepts such as sustainability and using them)
- Suggest (using scientific knowledge to more independently suggest different approaches/solutions etc.)
There is a need to apply this along with careful selection of scientific ideas that clearly vary in their complexity due to conceptual challenge and background skills etc. This system is used in connection with GCSE examination command words and the use of those for 'describing' or 'explaining' graphs can be particularly relevant in this topic given observation of past difficulties students have had tackling questions of this nature.


## Lesson design

The lesson was based around the interpretation of graphical information representing Felix Baumgartner's skydive in October 2012, as shown below. This meant that the graph was in an unfamiliar context and was made challenging through the presence of two lines based on higher level concepts.

[Graph from wired.com]

Students began and ended the lesson by looking at the graph and indicating their confidence against a number of statements, which are shown in the table later.

The activities prepared for the lesson included:

- A paired activity where each student received a graph that was not shown to their partner and each also received a prepared axis and grid. Students took turns to describe their graphs to each other whilst the partner attempted to draw the graph. They compared strengths and weaknesses afterwards.
- A comprehension style activity to interpret the graph lines mathematically. This activity was divided into simpler 'explain' questions such as giving Felix's speed or the speed of sound at different times. The 'analyse' questions delved deeper into mathematical understanding by comparing the speed of Felix and the speed of sound in relation to each other, such as how much faster than the speed of sound Felix was travelling. Some use of distance (area under the graph) was also used.
- An activity to discuss forces, which had already been taught during the year in relation to skydiving and parachutes.
- An activity to try to plot Felix's speed relative to sound as a new graph as a high level challenge.


## Results

Most students completed their confidence grids as intended, though some failed to complete all boxes in the second column. The results are shown below with a column indicating how many students showed improvement in their confidence in each skill.

| Graph skill | Confidence before |  |  | Confidence after |  |  | Improvement |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | (2) | - | () | (2) | - | (-) |  |
| I understand what an axis is and what they both show in this graph. |  | 2 | 24 |  | 2 | 23 |  |
| I understand what the line is telling about Felix. | 5 | 17 | 4 |  | 1 | 23 | 20 |
| I understand what the line is telling me about the speed of sound. | 5 | 14 | 7 |  | 3 | 19 | 15 |
| I could describe this graph to someone over the phone. | 10 | 16 | 1 |  | 2 | 23 | 24 |
| I could explain what the speed of Felix or sound is at any given time. | 8 | 11 | 6 |  | 5 | 18 | 16 |
| I could analyse how much faster or slower than the speed of sound Felix is falling at, at any time shown. | 7 | 19 |  |  | 9 | 14 | 17 |
| I could explain why this is by using my scientific knowledge of forces and waves. | 12 | 14 |  | 3 | 12 | 4 | 9 |
| I could draw a graph of how much faster or slower than sound Felix is falling using a piece of graph paper. | 12 | 12 | 2 | 4 | 9 | 5 | 8 |

;) = I am very confident that I could do this
: = I could have a go but am not totally confident
$\theta=$ I am not feeling confident that I could do this

The results shown in this table suggested that students were confident with axes before the lesson and so time spent on them in this lesson might have been an unproductive use of time. In some cases it would have been useful to quickly ensure this was the case with a quick question to check. The greatest improvement was seen in being able to describe the graph lines with strong but lesser improvement in explaining the graph, potentially highlighting the step-up in challenge as per Bloom's Taxonomy and the use of IDEALS. Some students felt that they would be more confident in the activities that were not completed during the lesson and this paves the way for a second lesson to check whether students' confidence was matched by the ability to complete these tasks and to address any issues in doing so.

## Evaluation

The confidence grids, teacher observations and questionnaires used with a selection of students (covering a range of initial and final confidence levels) were used to evaluate the success of the lesson.
Student confidence in their skills was low at the start of the lesson. Most students felt that they could understand the graph axes with only a small number confident that they could describe what the lines showed. However, some students did indicate that they could give Felix's speed at any time using the graph.
It was particularly interesting to discover that some students found mathematically explaining the graph lines most difficult and describing the graphs to each other least difficult whilst others gave a reverse answer for this. This possibly indicates the complexities of students' confidence and abilities being very varied even within one area of mathematics. Students did appear to focus well and engage with both activities.
Students said that they did find the lesson challenging (though work completing indicated that they met this challenge). Whilst students found different lesson activities most useful there were several comments expressing that reviewing answers as a class had been particularly useful, such as '...going over the questions as a class - helped me understand it...' - highlighting that whilst this lesson was student focused there was still a useful role for teacher led review of work, for at least some students.
Most students said that their confidence had improved with some also saying that it had helped them to remember skills, with comments such as 'It improved my confidence but also helped me develop skills I had forgotten.' This is important as it highlighted the need to refresh skills that may have previously been acquired but perhaps forgotten. This was clear for the working out of distance using area under the graph. This might have been better used as a further activity with a reminder and some more data to help guide students (perhaps in place of discussing forces).
One student did not feel her confidence had improved as she still did not understand the graph, although her confidence sheet indicates she did improve her confidence in some areas. She said '...I didn't really understand $i t$.' Clearly as with all lessons there is a potential need to give additional support to some students. In this instance a focused small group support might be required to help to build confidence in a tailored way. She had commented upon her ability relative to others and it is possible that confidence had been affected by her observations of her peers (saying '...I felt that I was the only one who didn't understand $i t$ ').
Some students also indicated that they would appreciate a wider array of data/information to support their understanding and this could indicate a need to push some students towards the more challenging creative activity that was not completed during the lesson, particularly for the most able. Again the use of the area under the graph and the redrawing of the graph could have been used, perhaps with more supporting and exemplar materials.

## Future teaching of this topic

The lesson appeared to work well for the activities carried out, however the area under the graph required a prompt for students and in a future lesson I would have prepared this and used it as the 'Link' activity, assuming it to be of a higher demand than reading from the graph. The link would be to the various mathematical skills such as area of a triangle and approximating area. A differentiated task with an easier line might be beneficial for some students.

The lesson prepared the way for a second hour lesson (or a longer lesson) to work on reinforcing the skills developed and then step forward to the more complex activities. With some students having indicated a confidence to attempt these more challenging tasks it would possibly be appropriate to give a short test to students to check that their ability correlates with their confidence in each skill, before setting some students to work on the suggested task and working with others (such as the student feeling that she was having difficulty) to resolve issues in the work undertaken so far.
Ensuring students could tackle each skill was flagged up by a student in her questionnaire as she said she would improve the lesson by making sure that '... there was a question we had to write down/answer to explain why it was possible.'

This lesson took place with a group of relatively high ability. Individual responses from those who are at a lower level in the group show that a different approach might be required with less able groups. Here a check on the ability to use axes would almost certainly be required and the graph used might be better if it focused on one line and concept, perhaps removing the speed of sound concept completely with a focus only on Felix's speed. The challenge here could still involve distance travelled. Finally the combined results, comments and knowledge of students' learning at the school show the importance of revisiting skills regularly to ensure that students recall methods and are confident in using them. This approach could easily be applied to other scenarios in science at Key Stage3 and 4.

## The Language of Mathematics in Science Teaching Approaches

The aim of this book is to provide teachers, and others with an interest in science education, with authentic accounts of classroom practice describing the teaching of mathematics in science lessons to pupils aged 11-16.

The main part of the book consists of eight teacher accounts of classroom activities designed to promote the use of some particular aspect of mathematics as part of learning science. Each account is accompanied by a narrative that sets the scene, provides a commentary on key points, and suggests a number of prompts to the reader to promote further reflection.

## In this book you will find:

- examples of collaboration between mathematics and science teachersscience learning activities designed to address specific mathematical ideas
$\square$ responses of individual pupils and groups to lesson activities


## $5\left(\frac{1}{3}\right.$ The Association nufor Science Education

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