RSC | Advancing the Chemical Sciences

Non-linear Cost Equation Example

There are many reasons why the cost equations may not be linear in form. It is often the case that costs increase more rapidly as full production capacity of the plan is reached. For example if a chemical plant is operating at full capacity all day every day, what will happen if there is a plant breakdown? Or, when will scheduled maintenance be undertaken? To manage such scenarios expensive overtime labour may have to be paid to keep the plant downtime to a minimum and production levels maintained. Therefore to reflect these issues assume that your total fixed cost for the ethylbenzene plant in the linear cost equation example is represented not by the constant (number) that you calculated but by a non-linear equation (number + $0.1\boldsymbol{x}^2$).

Answer the following questions:

(a) Determine the values in the empty table below:

Quantity manufactured (*x*) (t a^{-1})	**Total Fixed costs (*F*) (£ a^{-1})**	**Fixed costs per tonne of ethylbenzene (*f*) (£ t^{-1})**	**Total Variable costs (*V*) (£ a^{-1})**	**Variable costs per tonne of ethylbenzene (*v*) (£ t^{-1})**
0				
999				
1000				
1999				
2000				
2999				
3000				
3999				
4000				
4999				
5000				
5999				
6000				
6999				
7000				
7999				
8000				
8999				
9000				
9999				
10000				

(b) Draw a graph to show the variation of ***fixed costs per tonne (f)*** and ***variable costs per tonne (v)*** with the **quantity (*x*)** of ethylbenzene produced. Comment on your graphs.

(c) Prepare a Total Cost equation, which will show the variation of **total costs (*C*)** with **quantity (*x*)** of ethylbenzene produced. [Hint: this will be a quadratic equation]

(d) Prepare an Average Cost equation, which will show the variation of **average cost (*AC*)** with **quantity (*x*)** of ethylbenzene produced.

(e) Determine the numerical values in the empty table below.

Quantity manufactured (*x*) (t a^{-1})	**Total cost (C)**	**Total revenue (R)**	**Average cost (AC)**	**Marginal cost (MC)**	**Marginal revenue (MR)**	**Profit (P)**
0						
999						
1000						
1999						
2000						
2999						
3000						
3999						
4000						
4999						
5000						
5999						
6000						
6999						
7000						
7999						
8000						
8999						
9000						
9999						
10000						

(f) Prepare a **marginal cost (*MC*)** equation and a **marginal revenue (*MR*)** equation. [Hint: differentiate the appropriate equations]

(g) Draw a graph to show the variation of **total *fixed costs (F)***, **total *variable costs (V)*** and ***total costs (C)*** with **quantity (*x*)** manufactured. Comment on your graphs.

(h) Draw a graph to show the variation of ***marginal cost (MC), average costs (AC)***, ***marginal revenue (MR)*** and **profit (*P*)** with **quantity (*x*)** manufactured. Comment on your graphs.

(i) How much ethylbenzene should be manufactured in order to maximise the profit for the company? What is the minimum amount of ethylbenzene that should be manufactured in order to make a profit for the company?

(j) Comment on the consequences of a non-linear cost model for a production manager.

(k) Derive an equation to relate **marginal cost (*MC*)** with **marginal revenue (*MR*)** at the profit maximisation point. This is a vital relationship in market economics.

(l) Prove algebraically that the **average cost (*AC*)** curve must intersect the **marginal cost (*MC*)** curve at the minimum of the **average cost (*AC*)** curve.