## Linear Cost Equation Example

A chemical company operates a plant to produce ethylbenzene from benzene and ethene using a Friedel-Crafts alkylation process. A summary of information about the manufacturing plant is provided below:

| Item | Value |
| :--- | :--- |
| Labour costs | $£ 500,000 \mathrm{a}^{-1}$ |
| Benzene purchase price | $£ 600 \mathrm{t}^{-1}$ |
| Benzene usage | $0.75 \mathrm{t} \mathrm{t}^{-1}$ (of ethylbenzene produced) |
| Process water | $£ 10,000 \mathrm{a}^{-1}$ |
| Management costs | $£ 100,000 \mathrm{a}^{-1}$ |
| Marketing costs | $£ 200,000 \mathrm{a}^{-1}$ |
| Plant capacity | $100,000 \mathrm{t} \mathrm{a}^{-1}$ |
| Maintenance costs | $£ 50,000 \mathrm{a}^{-1}$ |
| Ethene purchase price | $£ 450 \mathrm{t}^{-1}$ |
| Ethene usage | $0.28 \mathrm{t} \mathrm{t}^{-1}($ of ethylbenzene produced) |
| Catalyst costs | $£ 100,000 \mathrm{a}^{-1}$ |
| R\&D costs | $£ 100,000 \mathrm{a}^{-1}$ |
| Ethylbenzene selling price | $£ 1600 \mathrm{t}^{-1}$ |
| Process energy costs | $£ 84,000 \mathrm{a}^{-1}$ |
| Depreciation of plant capital | $£ 500,000 \mathrm{a}^{-1}$ |
| Central Administration costs | $£ 50,000 \mathrm{a}^{-1}$ |

Answer the following questions:
(a) Categorise the cost items as either fixed or variable costs. Comment on any assumptions you may make.
(b) Determine the values in the empty table below:

| Quantity <br> manufactured <br> $(\boldsymbol{x})\left(\mathbf{t ~ a}^{-1}\right)$ | Total <br> Fixed <br> costs $(F)$ <br> $\left(£ \mathbf{a}^{-1}\right)$ | Fixed costs per <br> tonne of <br> ethylbenzene <br> $(f)\left(£ \mathrm{t}^{-1}\right)$ | Total Variable <br> costs $(V)$ <br> $\left(£ \mathbf{a}^{-1}\right)$ | Variable costs <br> per tonne of <br> ethylbenzene <br> $(v)\left(£ \mathrm{t}^{-1}\right)$ |
| :--- | :---: | :---: | :---: | :---: |
| 0 |  |  |  |  |
| 999 |  |  |  |  |
| 1000 |  |  |  |  |
| 1999 |  |  |  |  |
| 2000 |  |  |  |  |
| 2999 |  |  |  |  |
| 3000 |  |  |  |  |
| 3999 |  |  |  |  |
| 4000 |  |  |  |  |
| 4999 |  |  |  |  |
| 5000 |  |  |  |  |
| 5999 |  |  |  |  |
| 6000 |  |  |  |  |
| 6999 |  |  |  |  |


| 7000 |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- |
| 7999 |  |  |  |  |
| 8000 |  |  |  |  |
| 8999 |  |  |  |  |
| 9000 |  |  |  |  |
| 9999 |  |  |  |  |
| 10000 |  |  |  |  |

(c) Draw a graph to show the variation of average fixed costs per tonne (f) and average variable costs per tonne ( $v$ ) with the quantity $(x)$ of ethylbenzene produced. Comment on your graphs.
(d) Prepare a Total Cost equation, which will show the variation of total costs (C) with quantity $(x)$ of ethylbenzene produced.
(e) Prepare an Average Cost equation, which will show the variation of average cost (AC) with quantity $(x)$ of ethylbenzene produced.
(f) Determine the numerical values in the empty table below.

| Quantity <br> manufactured <br> $(\boldsymbol{x})\left(\mathbf{t ~ a}^{-1}\right)$ | Total <br> cost <br> (C) | Total <br> revenue <br> (R) | Average <br> cost <br> (AC) | Marginal <br> cost <br> (MC) | Marginal <br> revenue <br> (MR) | Profit <br> (P) |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 |  |  |  |  |  |  |
| 999 |  |  |  |  |  |  |
| 1000 |  |  |  |  |  |  |
| 1999 |  |  |  |  |  |  |
| 2000 |  |  |  |  |  |  |
| 2999 |  |  |  |  |  |  |
| 3000 |  |  |  |  |  |  |
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| 7000 |  |  |  |  |  |  |
| 7999 |  |  |  |  |  |  |
| 8000 |  |  |  |  |  |  |
| 9000 |  |  |  |  |  |  |
| 9999 |  |  |  |  |  |  |
| 10000 |  |  |  |  |  |  |

(g) Prepare a marginal cost (MC) equation and a marginal revenue (MR) equation
(h) Draw a graph to show the variation of total fixed costs (F), total variable costs (V) and total costs (C) with quantity ( $\mathbf{x}$ ) manufactured. Comment on your graphs.
(i) Draw a graph to show the variation of marginal cost (MC), average costs (AC), marginal revenue (MR) and profit $(P)$ with quantity $(x)$ manufactured. Comment on your graphs.
(j) How much ethylbenzene should be manufactured in order to maximise the profit ( $P$ ) for the company? What is the minimum amount of ethylbenzene that should be manufactured in order to make a profit for the company?

