Stage 2

Algebra and equations

ROYAL SOCIETY OF CHEMISTRY

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Introduction

In maths, you have learned how to substitute values into equations and solve for unknowns. In chemistry, we use the same algebra – with formulas for quantities like concentration, density and moles. This session builds on those maths skills and applies them to chemical calculations.

Learning objectives

1. Understand the algebra needed for balancing chemical equations.
2. Substitute numerical values into formulas and expressions using appropriate units.
3. Change the subject of an equation.
4. Solve for unknowns in chemical equations and stoichiometry problems.

Lesson question

During this session we will cover the mathematics skills needed to answer the following question:

Sulfuric acid reacts with sodium hydroxide to form sodium sulfate and water:

$$H_2SO_4 + NaOH \rightarrow Na_2SO_4 + H_2O$$

a) Balance the chemical equation.

b) A learner has 2.0 dm^3 of sulfuric acid solution containing 9.8 g of sulfuric acid. Calculate the concentration of the sulfuric acid in g/dm^3 using the equation:

$$\text{concentration} = \frac{\text{mass}}{\text{volume}}$$

c) The learner adds water to dilute the solution to a concentration of 2.45 g/dm^3. The mass of sulfuric acid stays the same.

Calculate the total volume of the diluted solution.

Learning from stage 1

Conservation of mass

In a chemical reaction, atoms are not created or destroyed – they are rearranged to form new substances. The total mass of the reactants equals the total mass of the products.

Balanced chemical equations

The number of atoms of each element must be the same on both sides of a chemical equation.

Quick check

10 g of calcium carbonate fully decomposes into calcium oxide and carbon dioxide. If the final mass of calcium oxide produced is 5.6 g, what is the mass of carbon dioxide?

What does balancing mean?

An unbalanced chemical equation shows us what reacts and what is produced. A balanced equation also tells us how much of each substance there is.

$$\mathbf{2}H_2 + O_2 \rightarrow \mathbf{2}H_2O$$

To balance an equation, we add **numbers** written before the formulas, these are called **coefficients**. We never change the **subscripts**, the small numbers written inside the formulas.

In the balanced symbol equation above we can see that the number of each type of atom is the same on both sides of the equation. Four hydrogen atoms and two oxygen atoms.

How do we balance equations?

Step 1: **list** all the elements in the equation.

Step 2: **count** the number of atoms of each element on each side.

Step 3: **balance** one element at a time – start with the one that appears in the fewest formulas. Adjust the coefficients.

Step 4: **check** all elements balance.

Step 5: **simplify**. Make sure the coefficients are the simplest whole number ratio.

Balancing equations

Worked example

Balance the following chemical equation:

$$Mg + O_2 \rightarrow MgO$$

Step 1: the elements are magnesium, Mg and oxygen, O.

Step 2: Mg is balanced: $1 \times Mg$ on each side of the equation.

O is not balanced: $2 \times O$ on the left-hand side, $1 \times O$ on the right.

Step 3: fix the oxygen by placing a 2 in front of MgO:

$$Mg + O_2 \rightarrow \mathbf{2}MgO$$

O is now balanced: $2 \times O$ on each side of the equation.

Mg is not balanced. Fix by placing a 2 in front of Mg:

$$\mathbf{2}Mg + O_2 \rightarrow \mathbf{2}MgO$$

Step 4: The equation is now balanced: $2 \times Mg$ and $2 \times O$ on each side.

Balancing equations practice

Practice

1. $Na + Cl_2 \rightarrow NaCl$
2. $H_2 + Cl_2 \rightarrow HCl$
3. $Fe + O_2 \rightarrow Fe_2O_3$
4. $N_2 + H_2 \rightarrow NH_3$
5. $C_2H_6 + O_2 \rightarrow CO_2 + H_2O$

Balancing equations practice – answers

Answers

1. $2Na + Cl_2 \rightarrow 2NaCl$
2. $H_2 + Cl_2 \rightarrow 2HCl$
3. $4Fe + 3O_2 \rightarrow 2Fe_2O_3$
4. $N_2 + 3H_2 \rightarrow 2NH_3$
5. $2C_2H_6 + 7O_2 \rightarrow 4CO_2 + 6H_2O$

Discuss balancing

Activity

Three learners are trying to balance the following equation:

$$Al + O_2 \rightarrow Al_2O_3$$

Which learner has balanced the equation correctly? Explain why the other two learners are wrong.

Learner A

$2Al + O_2 \rightarrow Al_2O_3$

Learner B

$Al + O_3 \rightarrow AlO_3$

Learner C

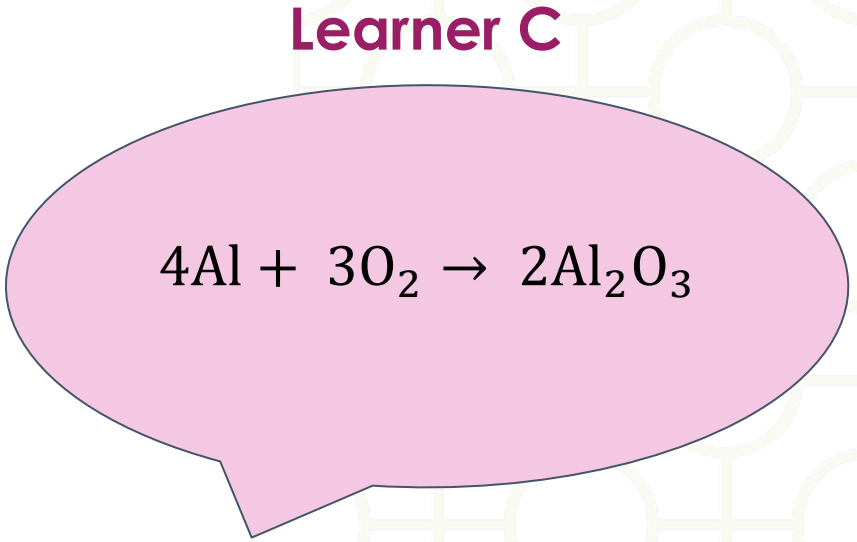
$4Al + 3O_2 \rightarrow 2Al_2O_3$

Discuss balancing – answer

Learner C is correct:
$4Al + 3O_2 \rightarrow 2Al_2O_3$
Al: $4 = 4$. O: $6 = 6$.

Learner A is incorrect because the equation is not fully balanced; Learner A balanced aluminium only.

Learner B is incorrect because they changed the formula of oxygen from O_2 to O_3.



Answer

Scientific formulas and algebraic equations

In science, we use formulas to describe relationships between quantities. These are algebraic equations – the letters represent physical quantities with units.

In maths you solve equations like $a = \frac{b}{c}$

In science, the letters have meaning like $c = \frac{m}{V}$, where:

c = concentration, m = mass and V = volume.

To solve science problems, we use the same algebra as maths: **substitute** the values we know, then **solve** for the unknown.

Substitution – you already do this in maths

Worked example

In maths:

If $y = \frac{x}{z}$,

find y when $x = 12$ and $z = 4$.

Substitute:

$$y = \frac{12}{4}$$

Solve:

$$y = 3$$

In science:

Find the density when mass = 12 g and volume = 4 cm^3.

$$d = \frac{m}{V}$$

Substitute:

$$d = \frac{12}{4}$$

Solve:

$$d = 3\ g/cm^3$$

Substitution

Worked example

A solution is made by dissolving 14.6 g of salt in water to make 2.0 dm^3 of solution. Calculate the concentration in g/dm^3.

Step 1: write the formula $\text{concentration} = \frac{\text{mass}}{\text{volume}}$

$$c = \frac{m}{V}$$

Step 2: substitute $c = \frac{14.6}{2.0}$

Step 3: calculate $c = 7.3$

Step 4: add units $c = 7.3\ g/dm^3$

Substitution practice

Practice

Use the four-step method to answer each question.

1. A block of metal has a mass of 54.0 g and a volume of 20.0 cm^3.

Calculate its density.

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$

2. A learner dissolves 8.0 g of sugar in water to make 0.5 dm^3 of solution.

Calculate the concentration in g/dm^3.

$$\text{concentration} = \frac{\text{mass}}{\text{volume}}$$

3. A solution has a concentration of 6.0 g/dm^3 and a volume of 0.5 dm^3.

Calculate the mass of solute dissolved.

$$\text{mass} = \text{concentration} \times \text{volume}$$

Substitution practice – answers

Answers

1. **Step 1: write** the formula:

   $\text{density} = \frac{\text{mass}}{\text{volume}}$, $d = \frac{m}{V}$

   Step 2: substitute: $\text{d} = \frac{54.0}{20.0}$

   Step 3: calculate: $d = 2.7$

   Step 4: add units: $d = 2.7\ \text{g/cm}^3$

2. **Step 1: write** the formula:

   $\text{concentration} = \frac{\text{mass}}{\text{volume}}$, $c = \frac{m}{V}$

   Step 2: substitute: $c = \frac{8.0}{0.5}$

   Step 3: calculate: $c = 16.0$

   Step 4: add units: $c = 16.0\ \text{g/dm}^3$

3. **Step 1: write** the formula:

   $\text{mass} = \text{concentration} \times \text{volume}$,

   $m = c \times V$

   Step 2: substitute: $m = 6.0 \times 0.5$

   Step 3: calculate: $m = 3.0$

   Step 4: add units: $m = 3.0\ \text{g}$

Changing the subject of an equation

In maths, you have learned to 'change the subject' of an equation. This means rearranging to get a different letter on its own.
The golden rule: whatever you do to one side, you must do to the other.

Example: If $a = \frac{b}{c}$, find c when $a = 5$ and $c = 4$.

First: **substitute** the values you know.

$$5 = \frac{b}{4}$$

Then: we want to **make b the subject**. It is being divided by 4, so we do the inverse − multiply both sides by 4:

$$5 \times 4 = \frac{b}{4} \times 4$$
$$5 \times 4 = b$$
$$b = 20$$

Changing the subject of an equation example

Worked example

In maths

If $a = \frac{b}{c}$, find c when $a = 5$ and $b = 20$.

Substitute: $5 = \frac{20}{c}$

Change the subject to c

Multiply both sides by c:

$$5 \times c = 20$$

Divide both sides by 5:

$$c = \frac{20}{5}$$

$$c = 4$$

In science

$$\text{concentration} = \frac{\text{mass}}{\text{volume}}$$

Find V when $c = 5$ g/dm^3 and $m = 20$ g.

Substitute: $5 = \frac{20}{V}$

Change the subject to V

Multiply both sides by V:

$$5 \times V = 20$$

Divide both sides by 5:

$$V = \frac{20}{5}$$

$$V = 4 \text{ dm}^3$$

Finding an unknown

Worked example

A solution has a concentration of 4.9 g/dm^3 and contains 9.8 g of solute. Find the volume.

Step 1: write the formula $\text{concentration} = \frac{\text{mass}}{\text{volume}}$

$c = \frac{m}{V}$

Step 2: substitute $4.9 = \frac{9.8}{V}$

Step 3: change the subject. Multiply both sides by V: $4.9 \times V = 9.8$

Divide both sides by 4.9: $V = \frac{9.8}{4.9}$

Step 4: calculate $V = 2.0$

Step 5: add units $V = 2.0\ \text{dm}^3$

Finding unknowns practice

Practice

1. A metal has a density of 2.7 g/cm^3 and a volume of 20.0 cm^3.

   Find the mass.

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$

2. A solution has a concentration of 5.0 g/dm^3 and a volume of 0.4 dm^3.

   Find the mass of solute dissolved.

$$\text{concentration} = \frac{\text{mass}}{\text{volume}}$$

3. A solution has a concentration of 8.0 g/dm^3 and contains 24.0 g of solute.

   Find the volume of solution.

$$\text{concentration} = \frac{\text{mass}}{\text{volume}}$$

Finding unknowns practice – answers

Answers

1. One calculation, unknown on top.

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$

$$2.7 = \frac{m}{20.0}$$

Multiply both sides by 2.0:

$$m = 2.7 \times 20.0$$

$$m = 54.0\ \text{g}$$

2. One calculation, unknown on top.

$$\text{concentration} = \frac{\text{mass}}{\text{volume}}$$

$$5.0 = \frac{m}{0.4}$$

Multiply both sides by 0.4:

$$m = 5.0 \times 0.4$$

$$m = 2.0\ \text{g}$$

3. Two calculations, unknown on bottom.

$$\text{concentration} = \frac{\text{mass}}{\text{volume}}$$

$$8.0 = \frac{24.0}{V}$$

Multiply both sides by V:

$$8.0 \times V = 24.0$$

Divide both sides by 8.0: $V = \frac{24.0}{8.0}$

$$V = 3.0\ \text{dm}^3$$

Putting it all together

Many chemistry questions combine balancing, substitution and rearranging. Here is a general approach:

1. **Balance the equation** – use coefficients to make sure atoms balance on both sides
2. **Choose the right formula** – which equation links the quantities in the question?
3. **Substitute the known values** – replace letters with numbers
4. **Rearrange** and **calculate** – use inverse operations to find the unknown
5. **Add units** – remember to include the correct units

Note: some examples will also need unit conversions too. It is normally easier to convert units before substituting numbers into the equation.

Discussing algebra in chemistry

Three learners make a solution by dissolving 6.0 g of salt in water to make 1.5 dm^3. They want to find the concentration.

$$\text{concentration} = \frac{\text{mass}}{\text{volume}}$$

Which learner is correct? Explain why the other two learners are wrong.

Learner A

$$c = \frac{m}{V}$$
$$c = \frac{6.0}{1.5}$$
$$c = 4$$

Learner B

$$c = \frac{m}{V}$$
$$c = \frac{1.5}{6.0}$$
$$c = 0.25\ \text{g/dm}^3$$

Learner C

$$c = \frac{m}{V}$$
$$c = \frac{6.0}{1.5}$$
$$c = 4.0\ \text{g/dm}^3$$

Discussing algebra in chemistry – answer

Answer

Learner C is correct: $c = \frac{6.0}{1.5} = 4.0\ \text{g/dm}^3$

Learner A is incorrect because they forgot the units. The calculation is correct $(6.0 \div 1.5 = 4)$ but without units the answer is incomplete.

Learner B is incorrect because they divided the wrong way round. They calculated $\text{volume} \div \text{mass}\ (1.5 \div 6.0 = 0.25)$ instead of $\text{mass} \div \text{volume}\ (6.0 \div 1.5 = 4.0)$.

Learner C

$$c = \frac{m}{V}$$

$$c = \frac{6.0}{1.5}$$

$$c = 4.0\ \text{g/dm}^3$$

Returning to the lesson question

Sulfuric acid reacts with sodium hydroxide to form sodium sulfate and water:

$$H_2SO_4 + NaOH \rightarrow Na_2SO_4 + H_2O$$

a) Balance the chemical equation.

b) A learner has 2.0 dm^3 of sulfuric acid solution containing 9.8 g of sulfuric acid. Calculate the concentration of the sulfuric acid in g/dm^3 using the equation:

$$\text{concentration} = \frac{\text{mass}}{\text{volume}}$$

c) The learner adds water to dilute the solution to a concentration of 2.45 g/dm^3. The mass of sulfuric acid stays the same.

Calculate the total volume of the diluted solution.

Returning to the lesson question – answers

Answers

a) **(1)** balance the chemical equation: $H_2SO_4 + 2NaOH \rightarrow Na_2SO_4 + 2H_2O$

b) **(2)** write the formula: $\text{concentration} = \frac{\text{mass}}{\text{volume}}$, $c = \frac{m}{V}$

(3) substitute: $c = \frac{9.8}{2.0}$

(4) calculate: $c = 4.9$

(5) add units: $c = 4.9\ \text{g/dm}^3$

c) The mass of sulfuric acid stays the same (9.8 g). Only water is added. (No equation to balance.)

(2) write the formula: $\text{concentration} = \frac{\text{mass}}{\text{volume}}$, $c = \frac{m}{V}$

(3) substitute: $2.45 = \frac{9.8}{\text{V}}$

(4) Rearrange and calculate: change the subject (unknown on the bottom, two calculations):

Multiply both sides by V: $2.45 \times V = 9.8$

Divide both sides by 2.45: $V = \frac{9.8}{2.45}$ $V = 4.0$

(5) add units: $V = 4.0\ \text{dm}^3$