

Stage 2 ★ ★

Creating graphs



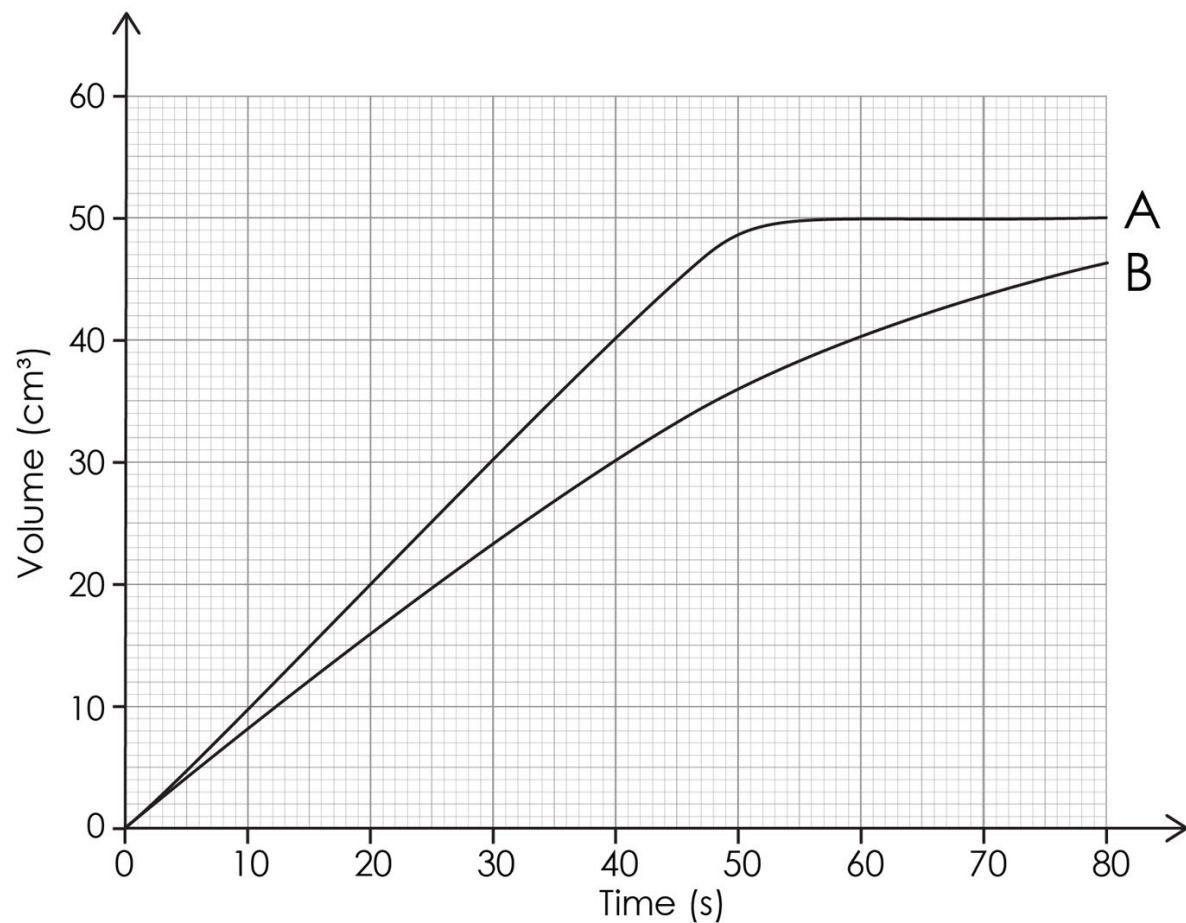
Introduction

In this lesson we will cover the maths skills needed to answer two graph questions. The first focuses on calculating the gradient of a slope and the second on using a tangent to calculate the gradient of a curved line.

Learning objectives

1. Calculate the gradient of a slope.
2. Describe mathematical relationships between two variables.
3. Find the rate of a reaction.
4. Use a tangent to find the gradient at a point on a curve.

Lesson question 1

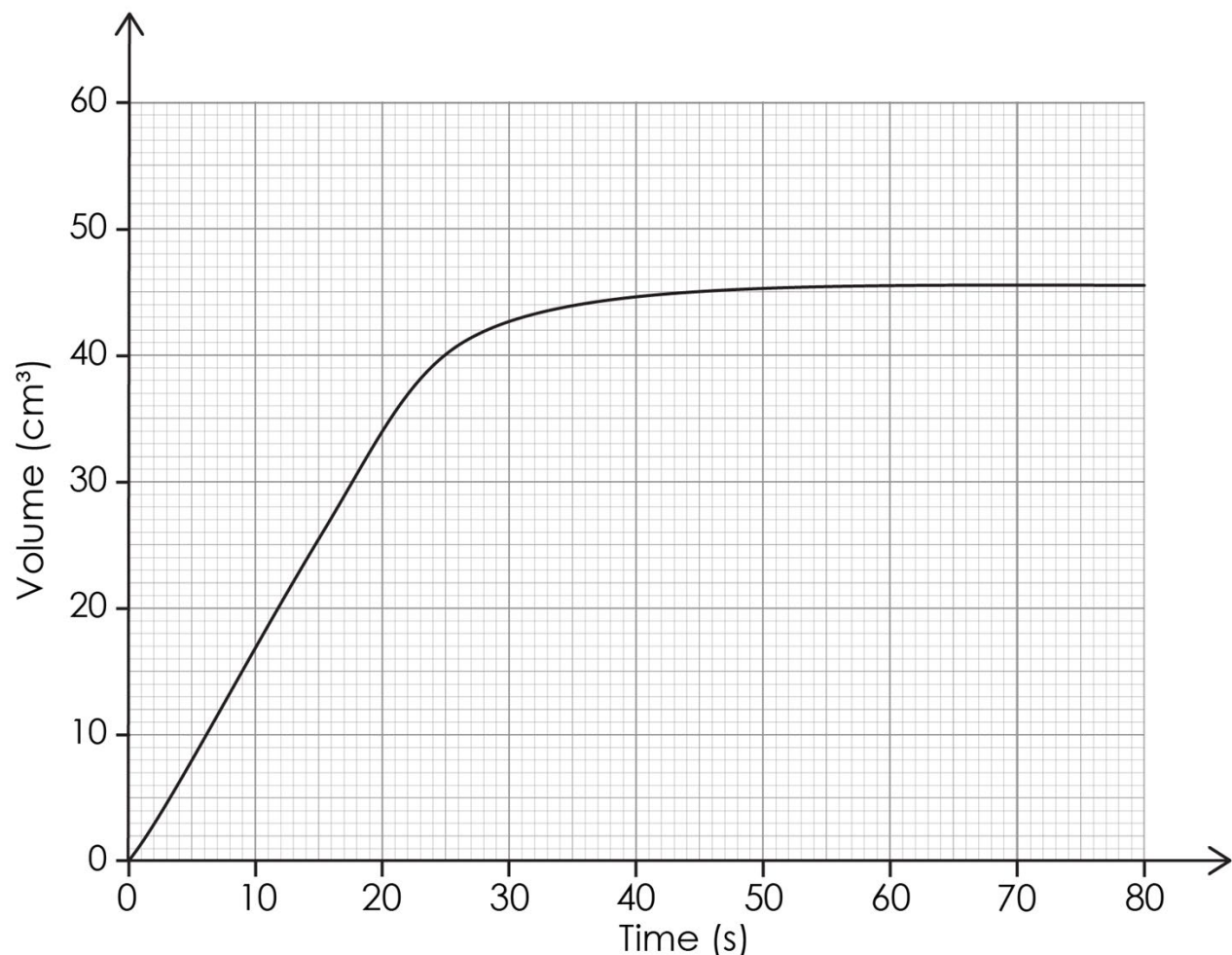


Imagine you are investigating the effect of changing the size of calcium carbonate lumps on the rate of the reaction with hydrochloric acid.

Compare the **mean rate** of reaction between **0 and 40 seconds** for reaction A and reaction B.

Lesson question 2

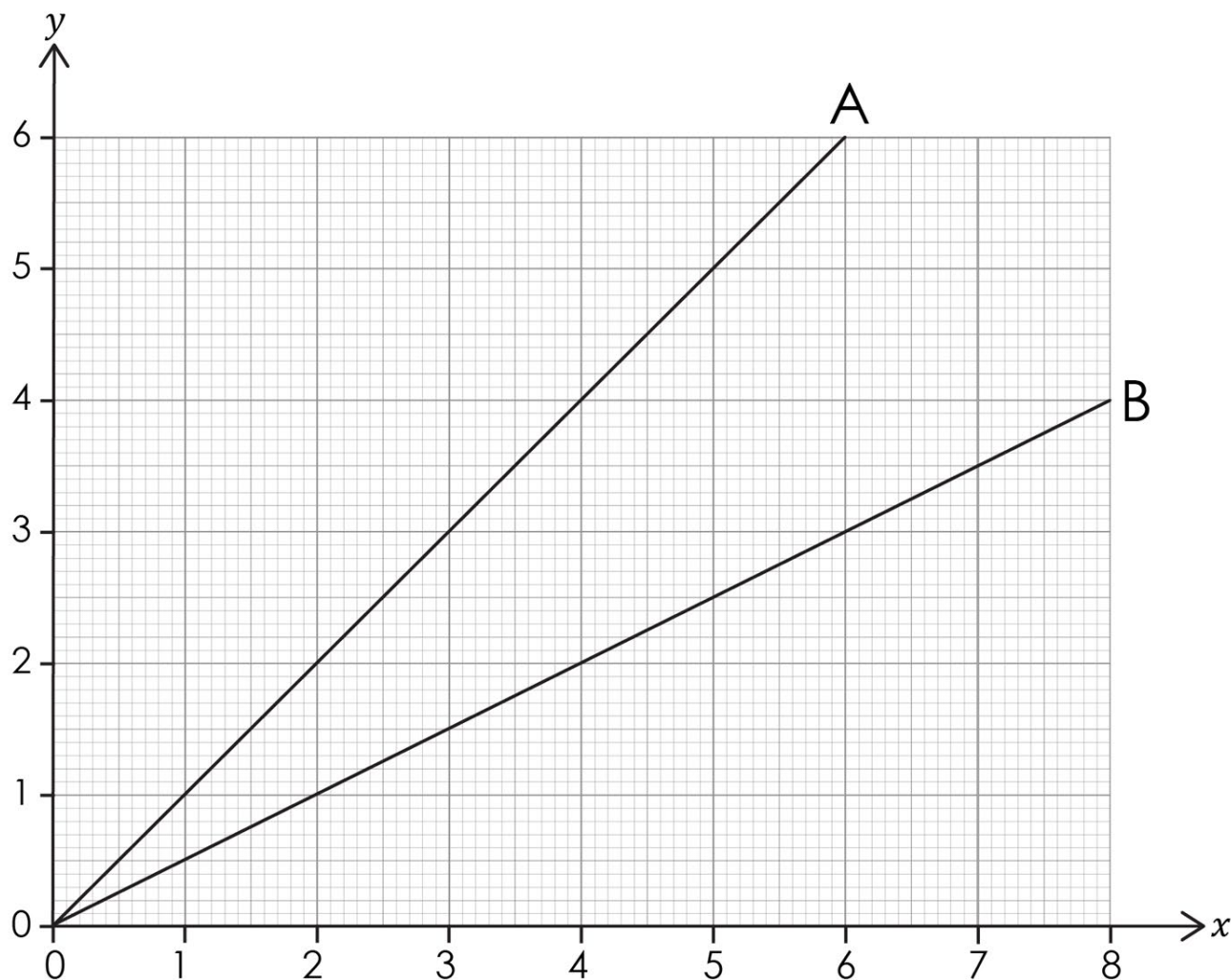
Let's look at an example that involves finding the gradient of a curved line at a specific point:



Imagine you are investigating the effect of changing the size of calcium carbonate lumps on the rate of the reaction with hydrochloric acid.

Determine the **rate of reaction** at **25 seconds**.

Describing the gradient of a slope



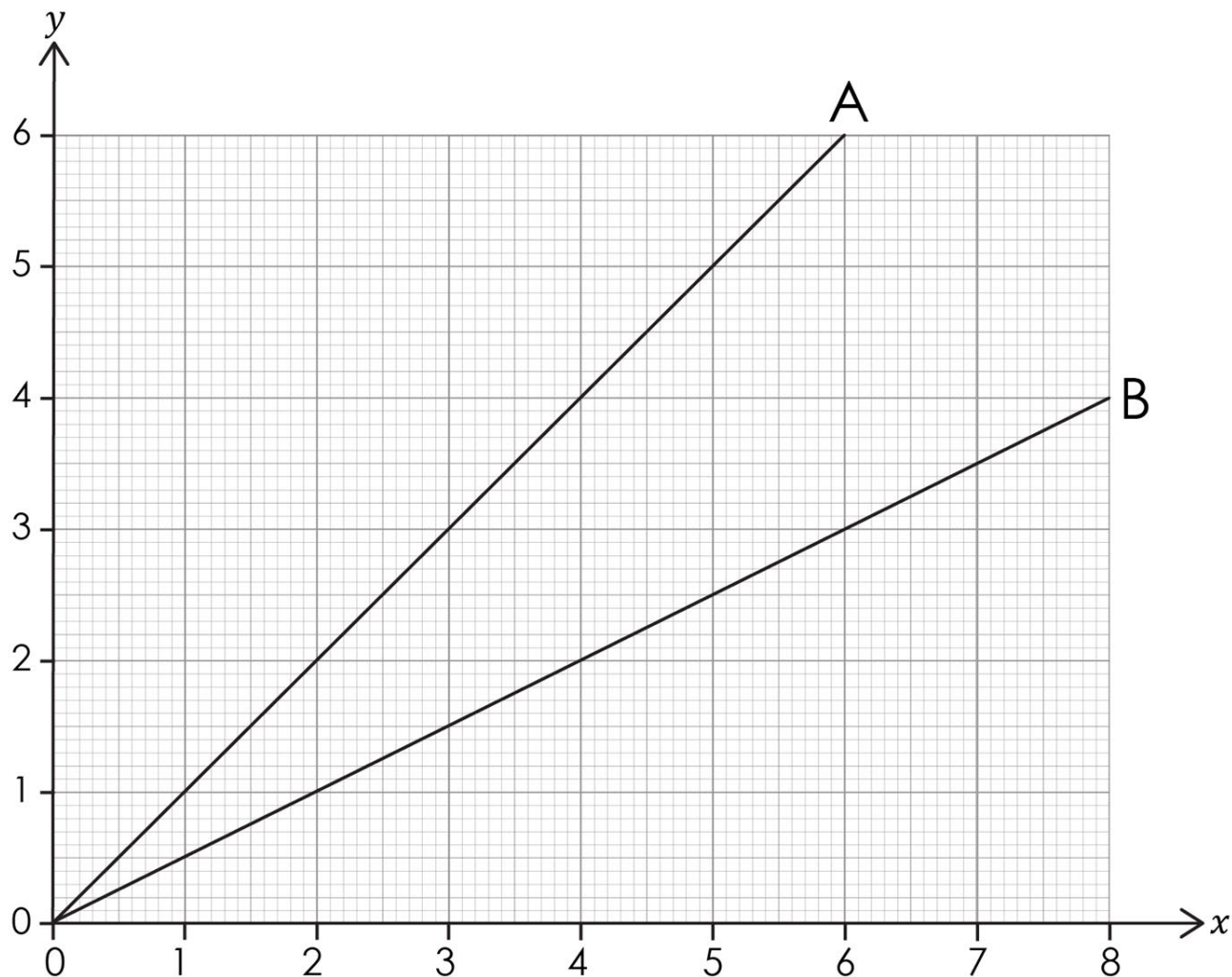
The gradient of a slope describes its steepness.

The greater a gradient, the steeper the slope.

A steeper slope means a faster rate.

- a) Which of these lines has a higher rate?
- b) Explain your answer.

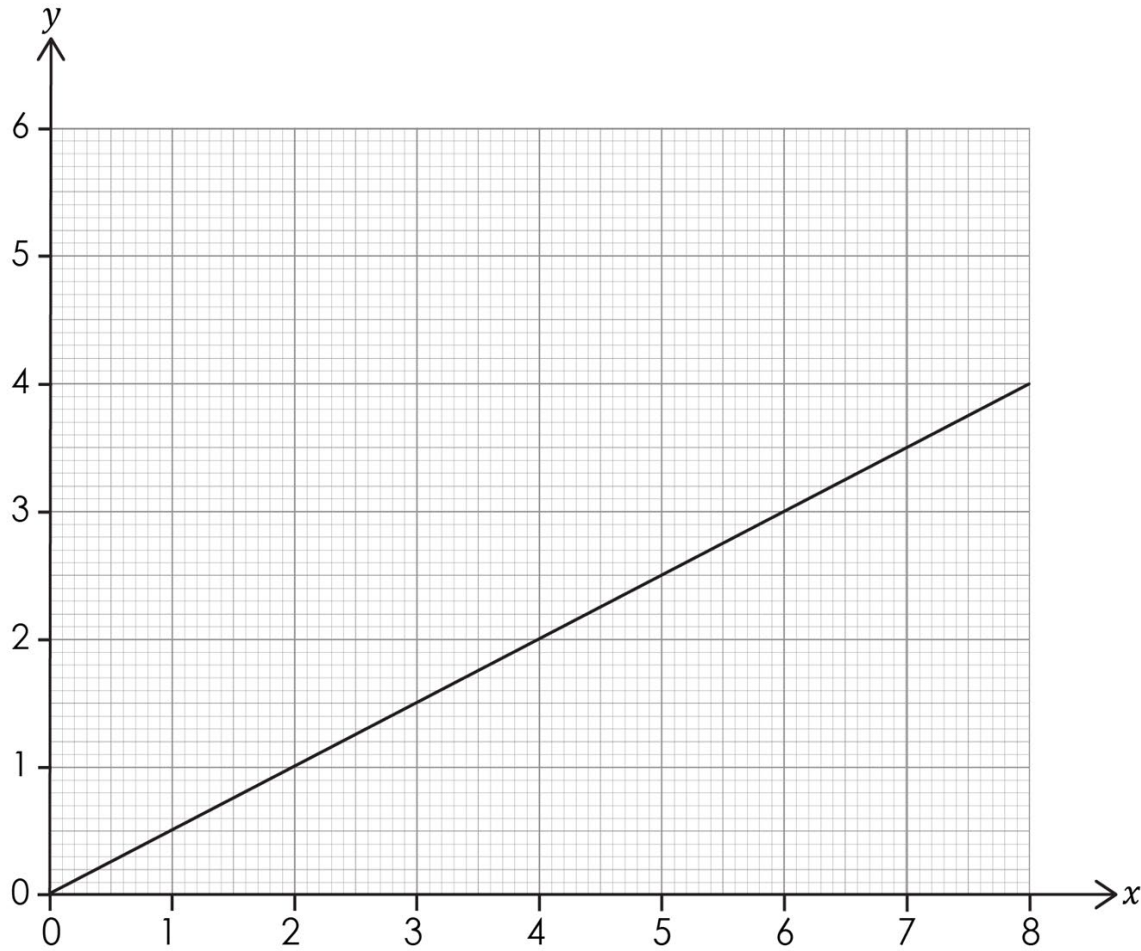
Describing the gradient of a slope – answer



- a) Line A has the higher rate because the gradient is steeper.
- b) As x increases y increases.



Calculating the gradient of a slope



Step 1: Identify two points on the line.

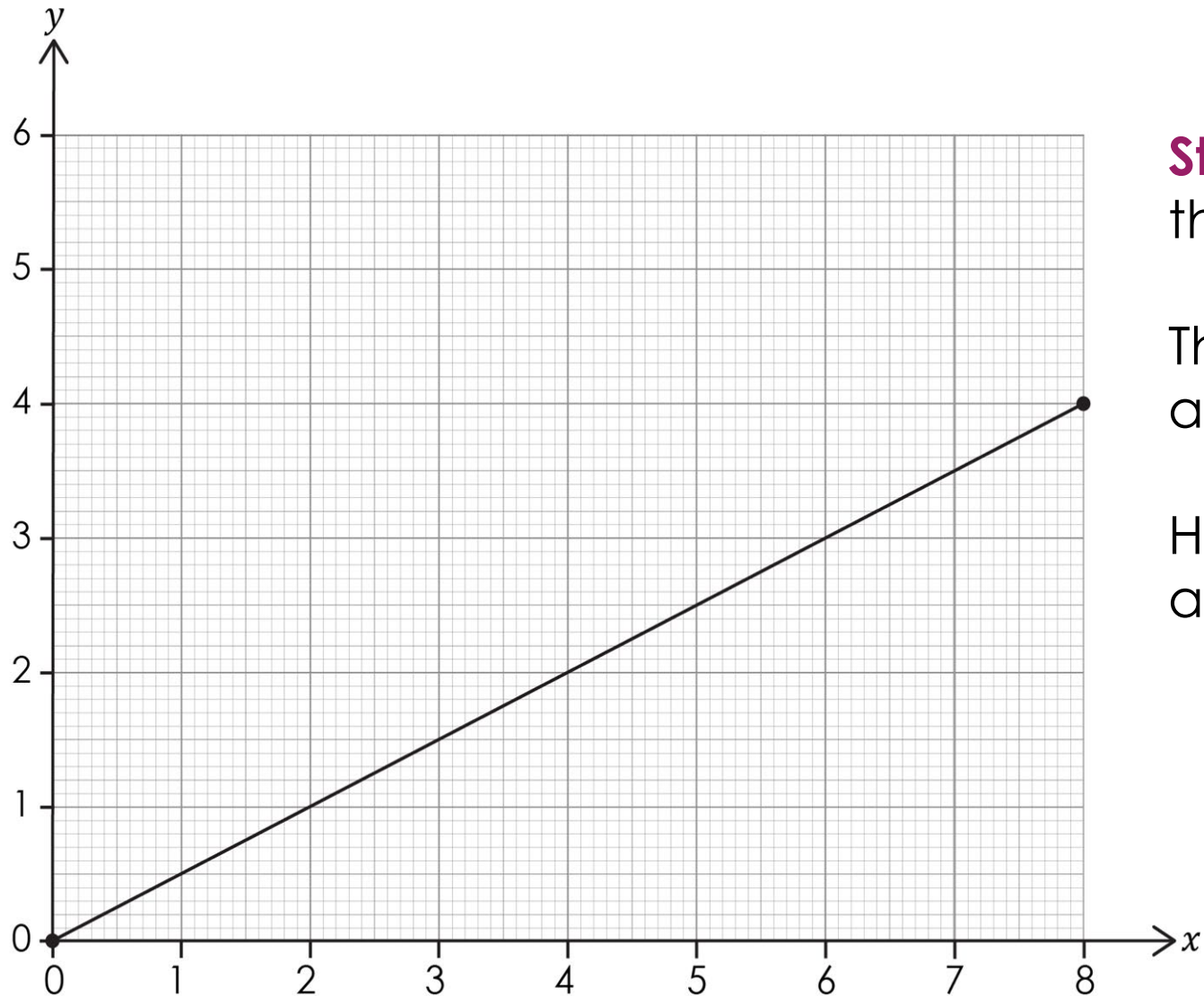
Step 2: Find the difference in y and the difference in x .

Step 3: Substitute the values into the following equation:

$$\text{gradient} = \frac{\text{difference in } y}{\text{difference in } x}$$



Calculating the gradient of a slope – step 1



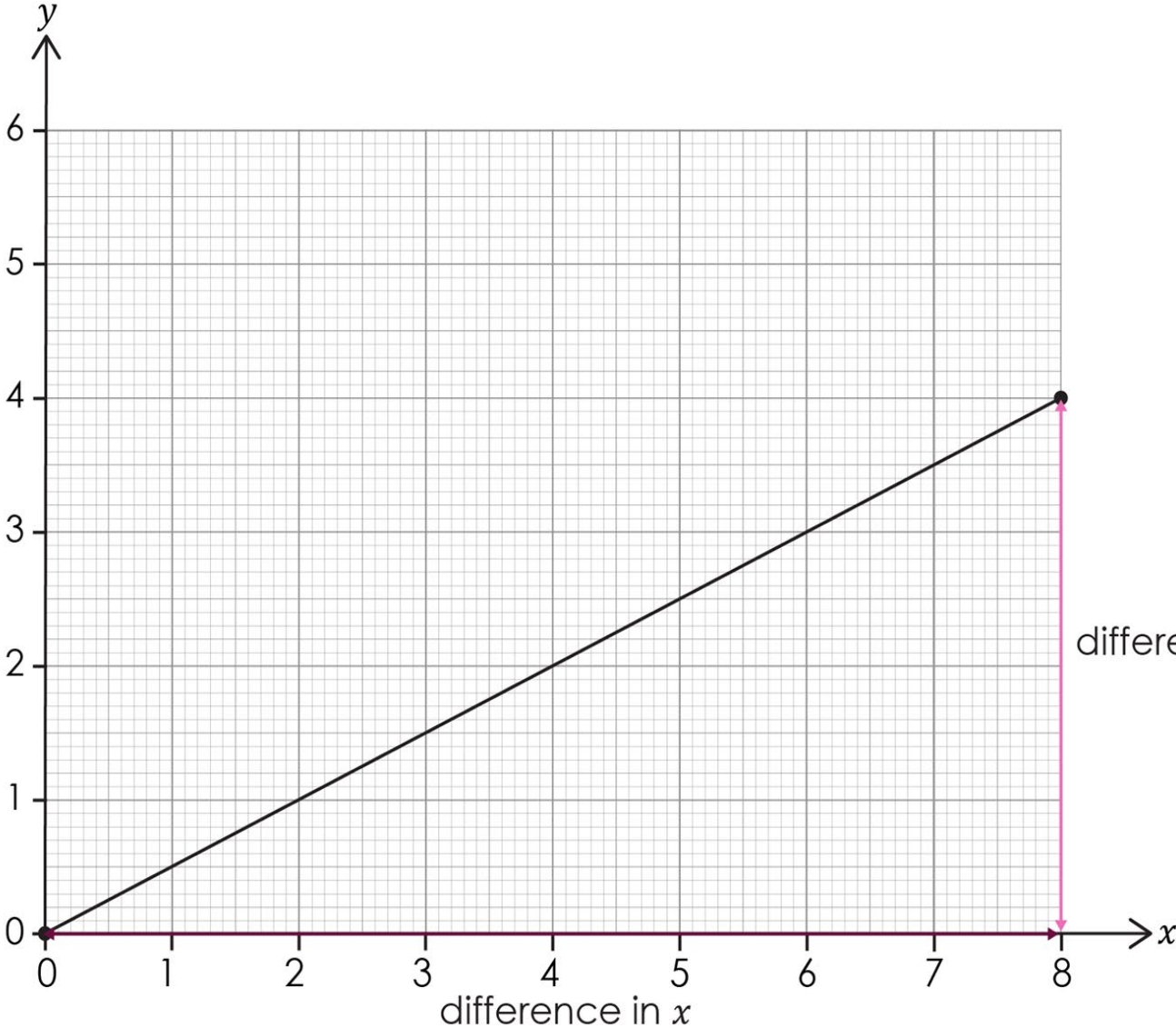
Step 1: Identify two points on the line.

This works best if the numbers are whole numbers.

Here we have chosen the start and the end of the line.



Calculating the gradient of a slope – step 2



Step 2: Find the difference in y and the difference in x .

difference in $y = 4 - 0 = 4$

difference in $x = 8 - 0 = 8$



Calculating the gradient of a slope – step 3

Step 3: Substitute the values into the equation.

difference in $y = 4$

difference in $x = 8$

$$\text{gradient} = \frac{\text{difference in } y}{\text{difference in } x}$$

$$\text{gradient} = \frac{4}{8}$$

$$\text{gradient} = 0.5$$

Differences between mathematics and science

In maths you are expected to be able to:

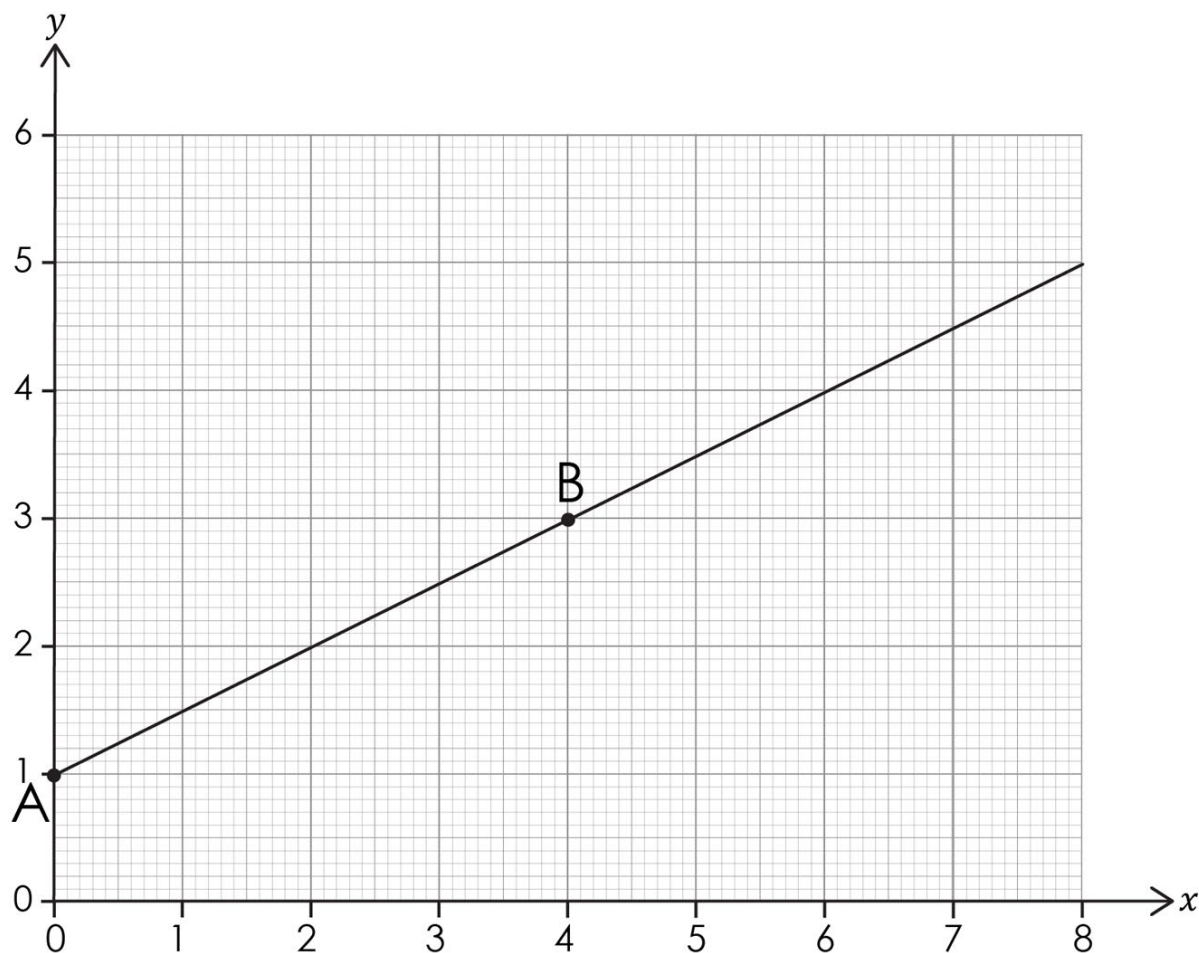
- interpret the gradient of a straight-line graph as a rate of change
- calculate the gradient of a straight line

This is often carried out using the same method as we use in science, but you can find the gradient using coordinates on the line as well.

Let's look at how this works.

Finding the gradient using coordinates – step 1

Step 1: Choose two coordinates on the line.



Remember – any straight line has a constant gradient.

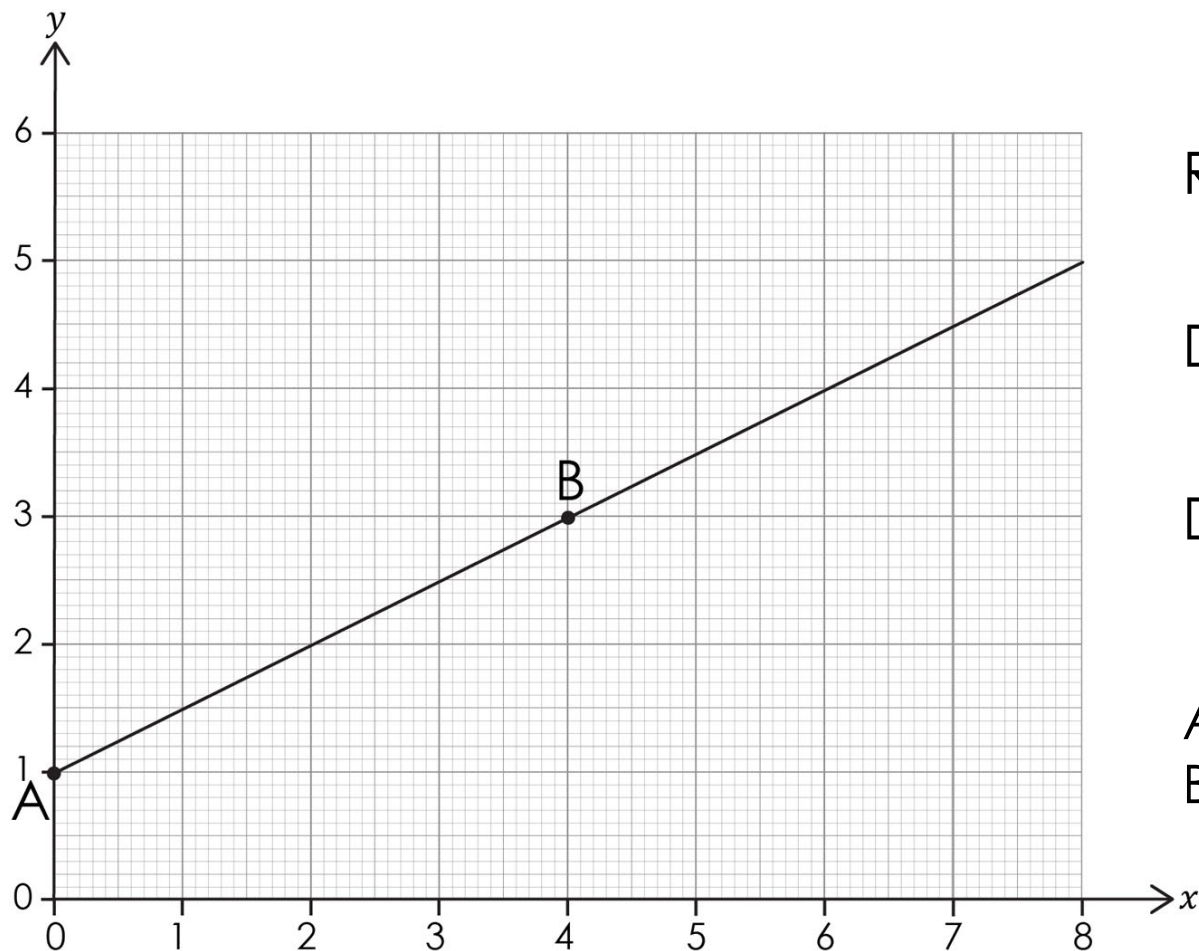
This means we can pick any two points, or coordinates, along its length to find the gradient.

$$A = (0, 1)$$

$$B = (4, 3)$$

Finding the gradient using coordinates – step 2

Step 2: find the difference in the y coordinates and the x coordinates.



Remember – a coordinate is (x, y)

Difference in y coordinates:

$$3 - 1 = 2$$

Difference in x coordinates:

$$4 - 0 = 4$$

$$A = (0, 1)$$

$$B = (4, 3)$$

Finding the gradient using coordinates – step 3

Step 3: divide the difference in the y coordinates by the difference in the difference in x coordinates.

$$\text{gradient} = \frac{\text{difference in } y \text{ coordinates}}{\text{difference in } x \text{ coordinates}}$$

$$\text{gradient} = \frac{2}{4}$$

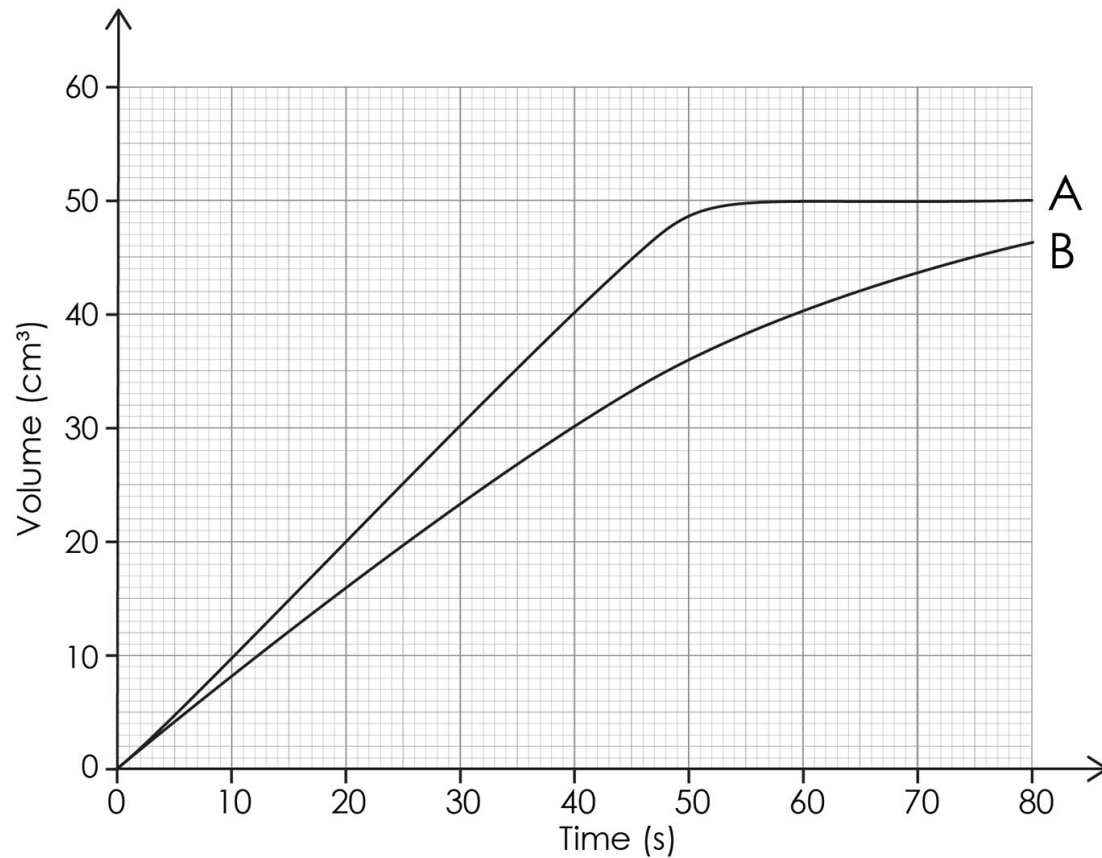
$$\text{gradient} = 0.5$$

Remember – each data point on a graph has an x coordinate and a y coordinate (x, y).

In science, the x coordinate is the **independent variable** and the y coordinate is the **dependent variable**.

Returning to lesson question 1

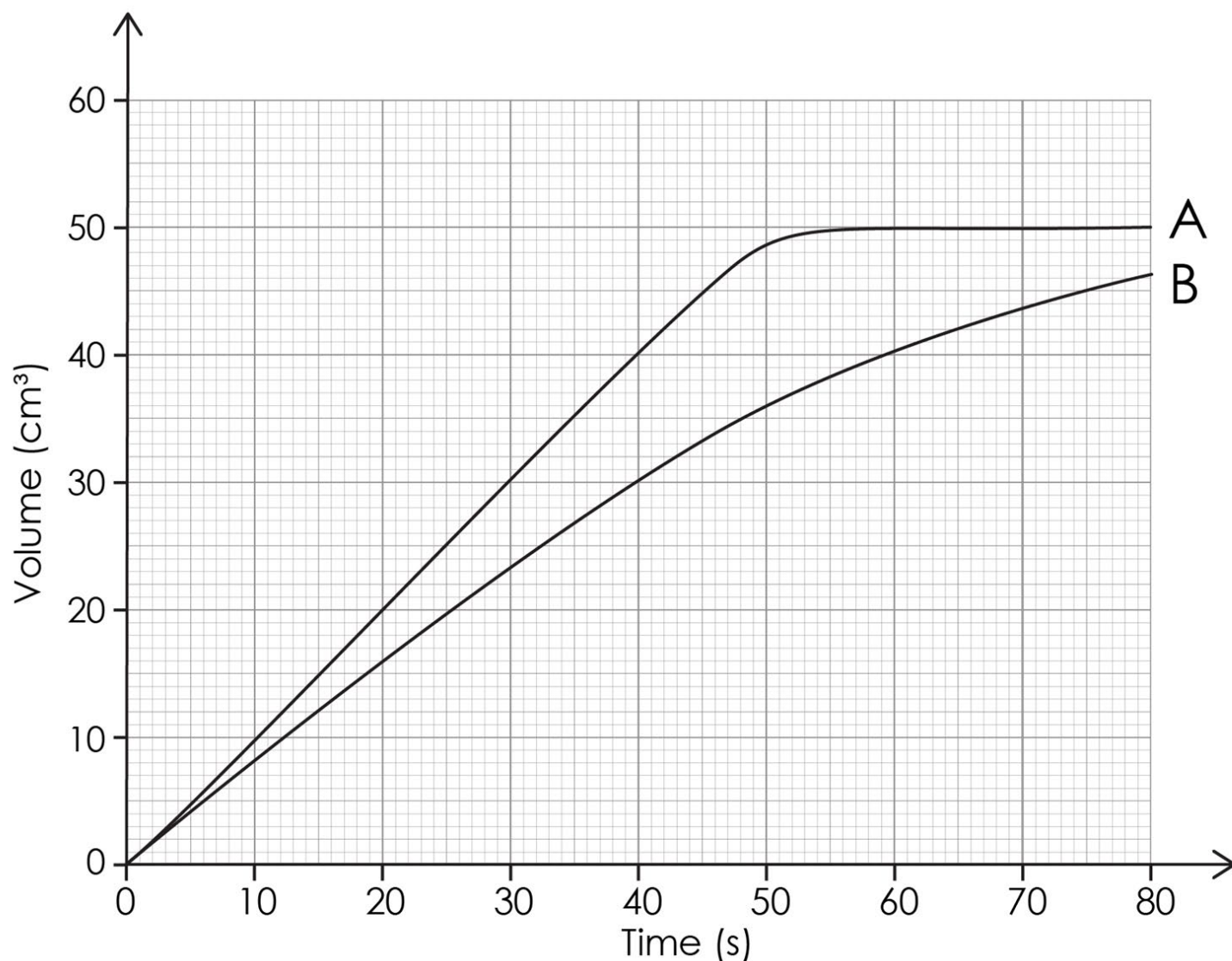
Let's go back to the example from the beginning of this lesson on calculating the gradient of a slope:



Imagine you are investigating the effect of changing the size of calcium carbonate lumps on the rate of the reaction with hydrochloric acid.

Compare the **mean rate** of reaction between **0 and 40 seconds** for reaction A and reaction B.

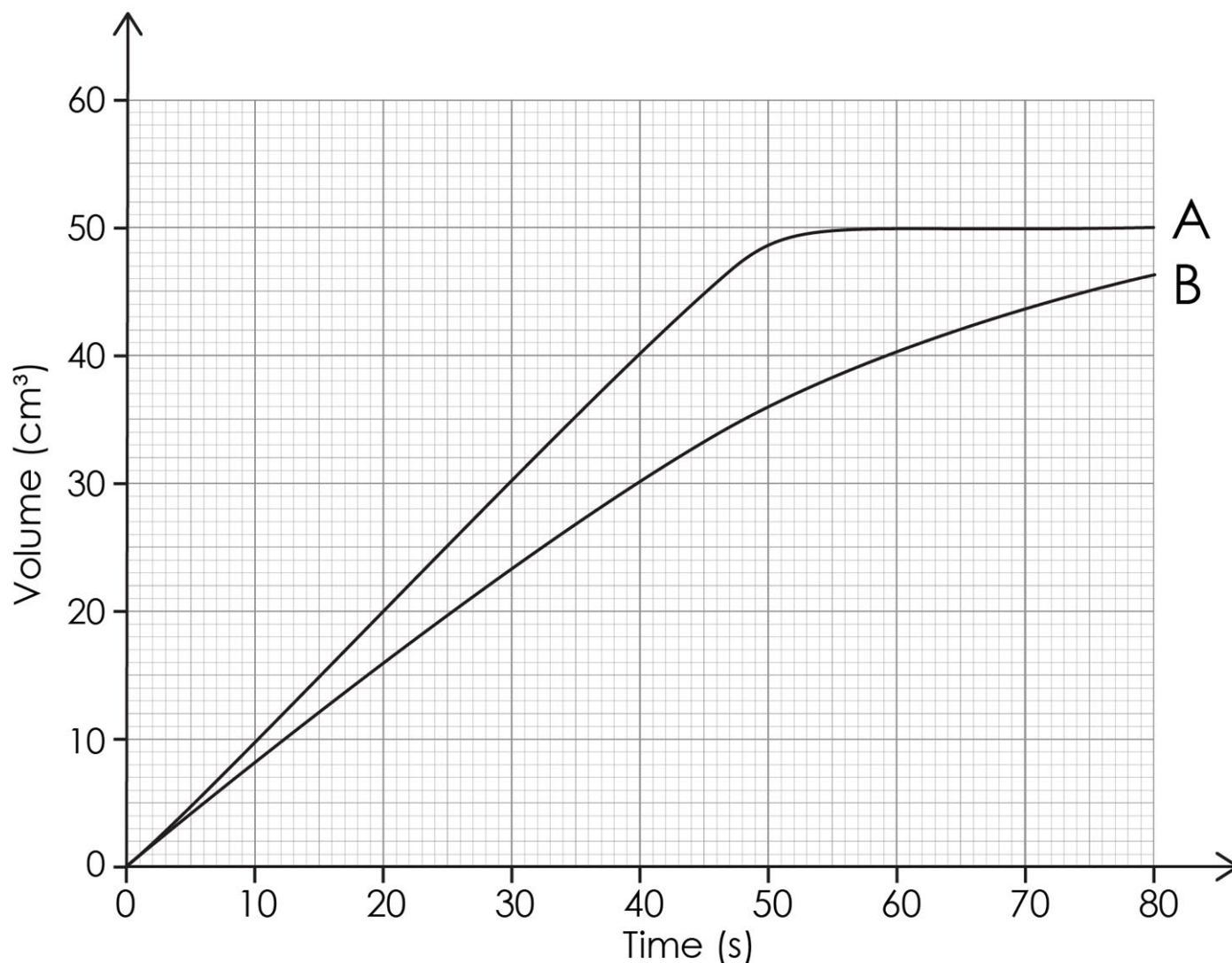
Describing the gradient of a slope – activity



Which reaction do you think is faster?

How do you know?

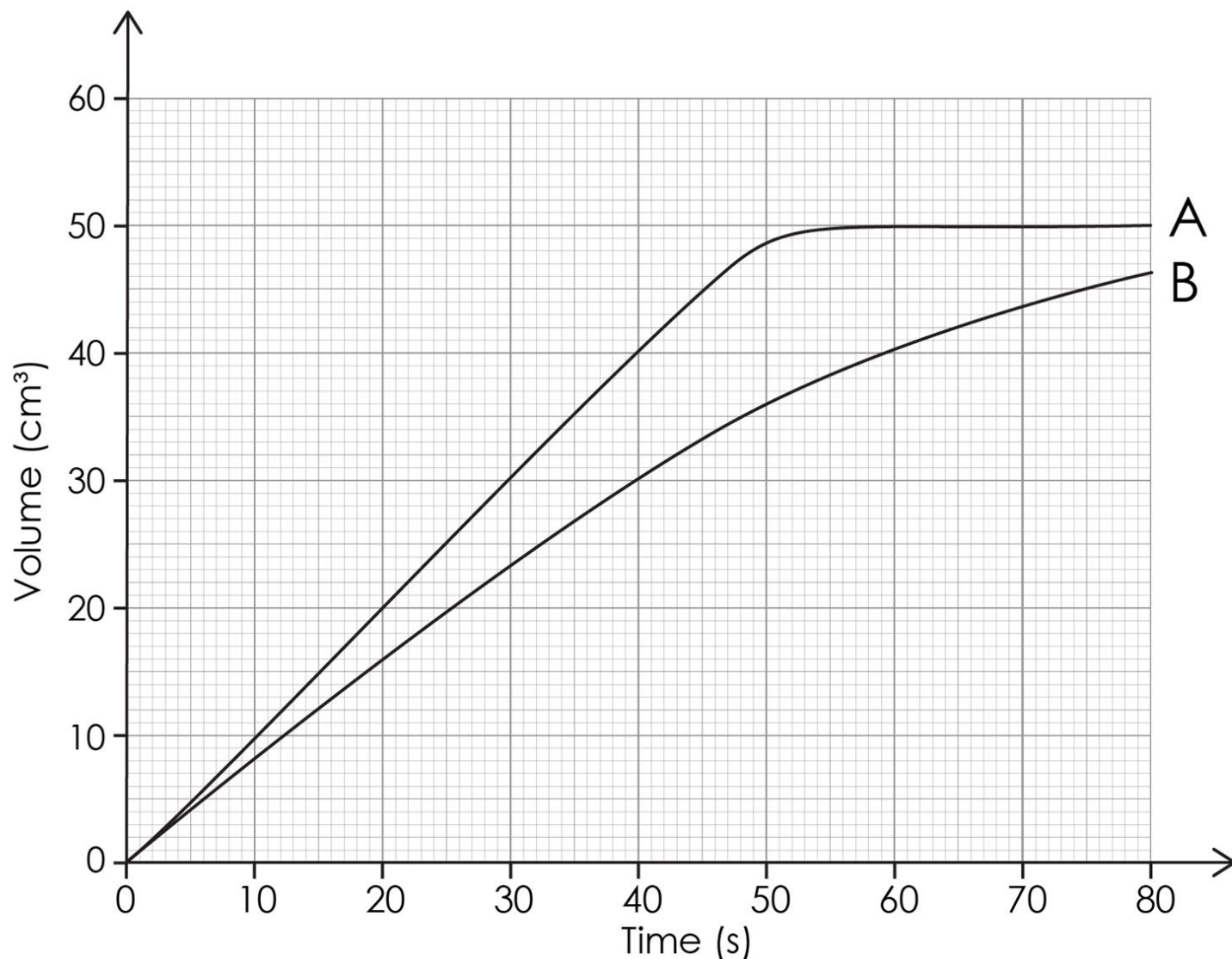
Describing the gradient of a slope – activity – answer



Reaction A is faster because:

- the line of best fit is steeper/the gradient is higher
- less time is needed to collect the same volume of gas/more gas is collected in the same amount of time.

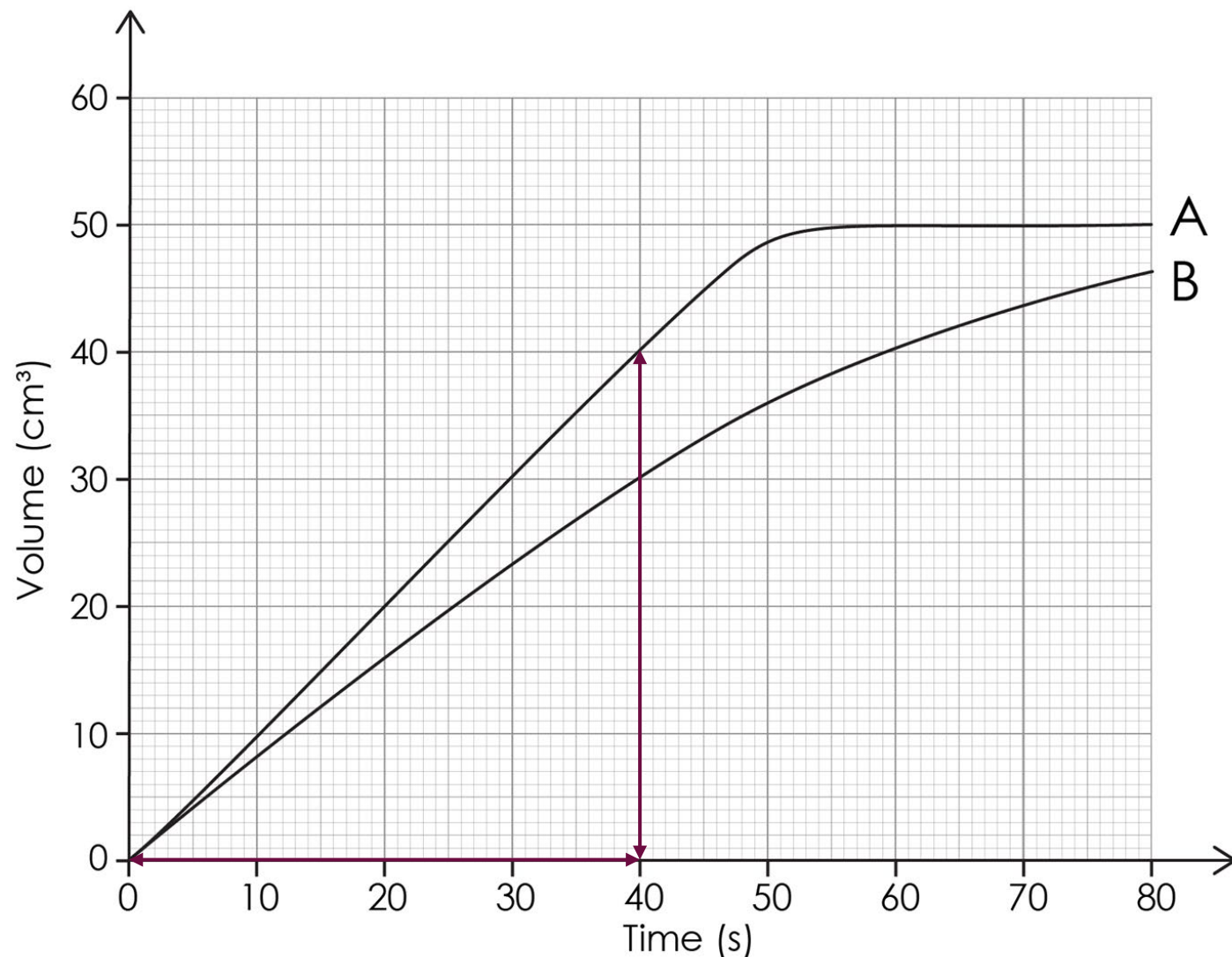
Calculating the gradient of a slope – activity



Calculate the mean rate of reaction between 0 and 40 seconds for:

- a) reaction A
- b) reaction B

Calculating the gradient of a slope – answer a



Calculate the mean rate of reaction between **0 and 40 seconds** for:

a) reaction A

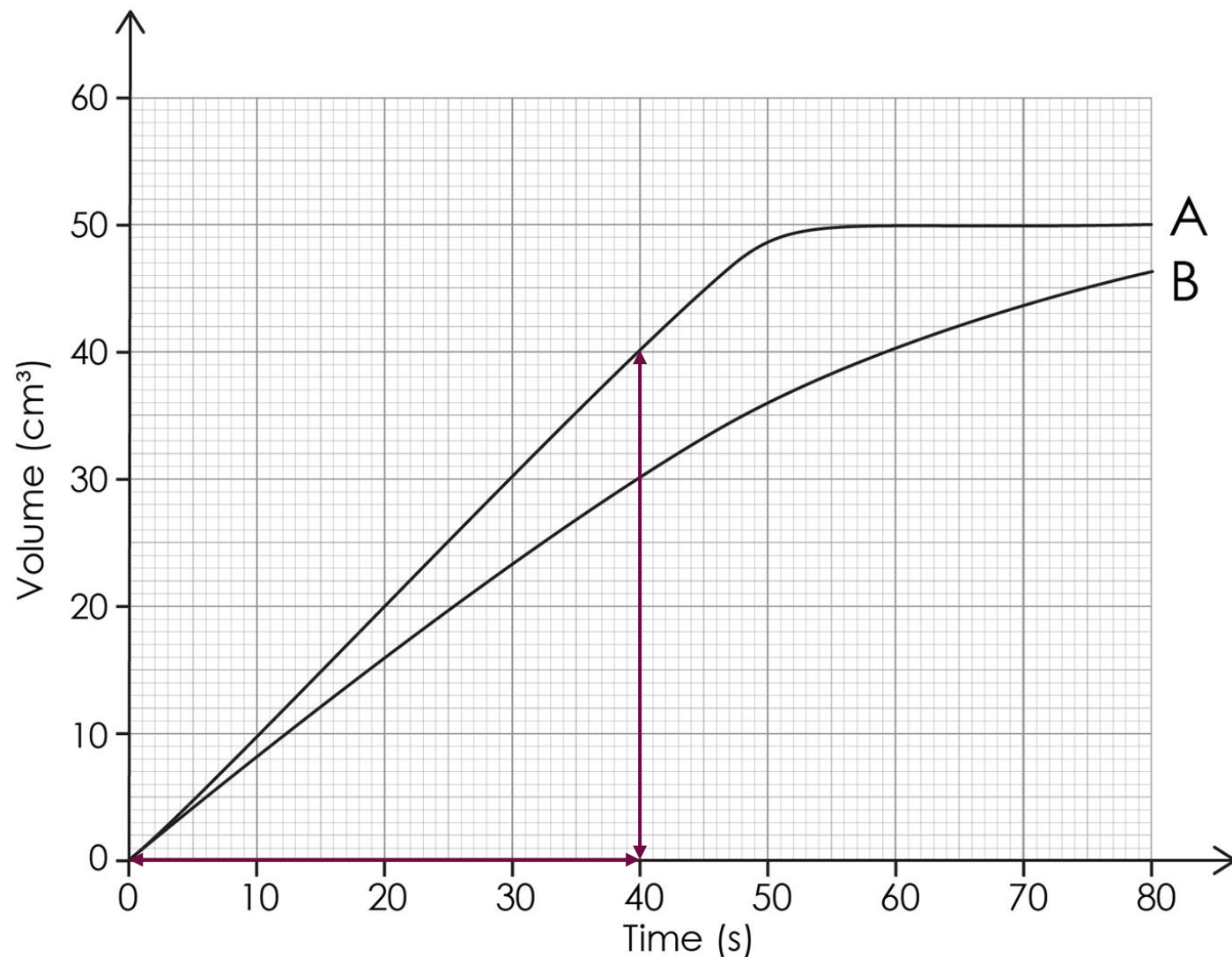
$$\text{difference in } y = 40 - 0 = 40$$

$$\text{difference in } x = 40 - 0 = 40$$

$$\text{gradient} = \frac{40}{40}$$

$$\text{gradient} = 1$$

Deriving the units – answer a continued



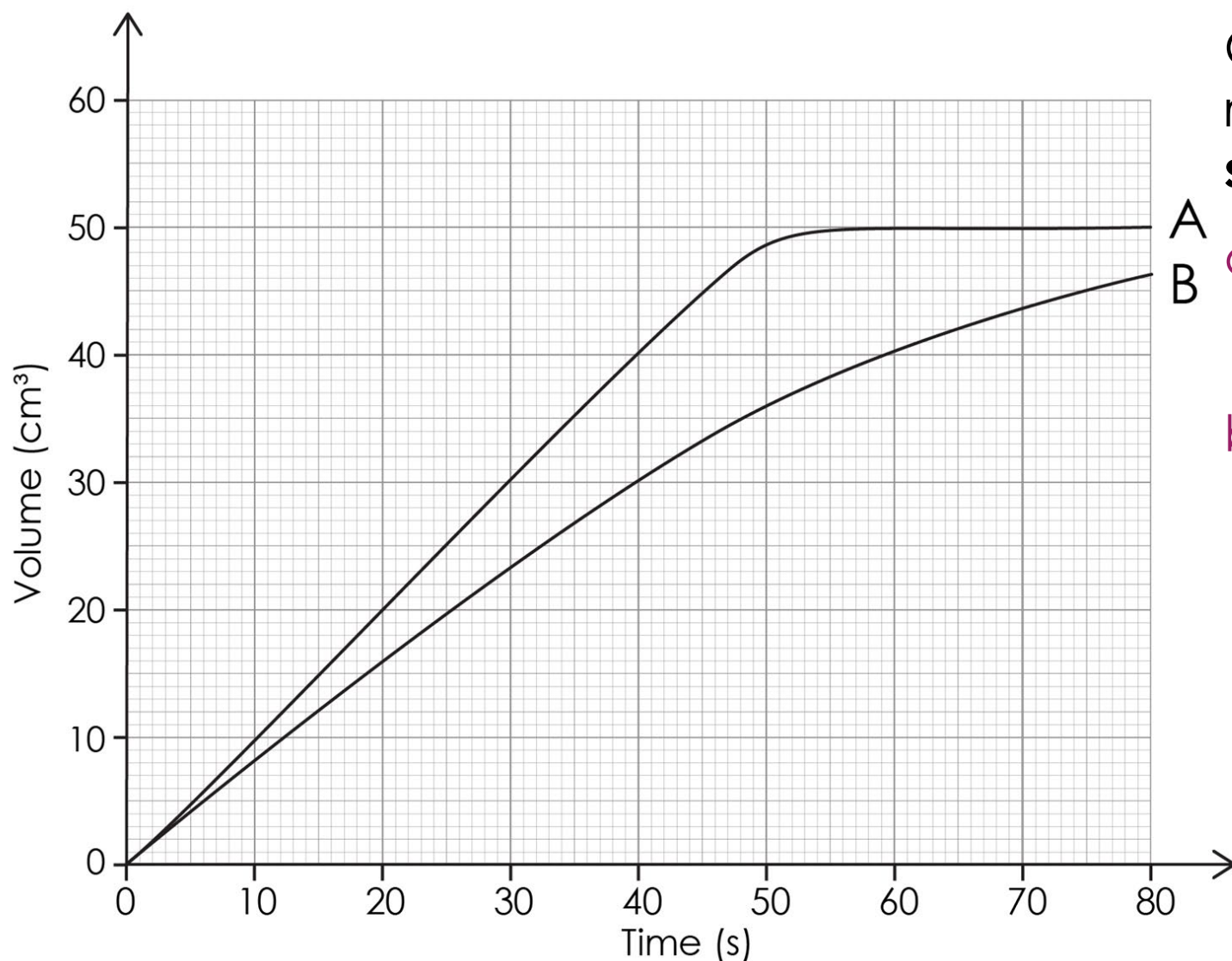
We can use the same equation to **derive** (work out) the units for the gradient.

$$\text{units} = \frac{\text{units on } y \text{ axis}}{\text{units on } x \text{ axis}}$$

$$\text{units} = \frac{\text{cm}^3}{\text{s}}$$

$$\text{units} = \text{cm}^3/\text{s}$$

Calculating the gradient of a slope – part b

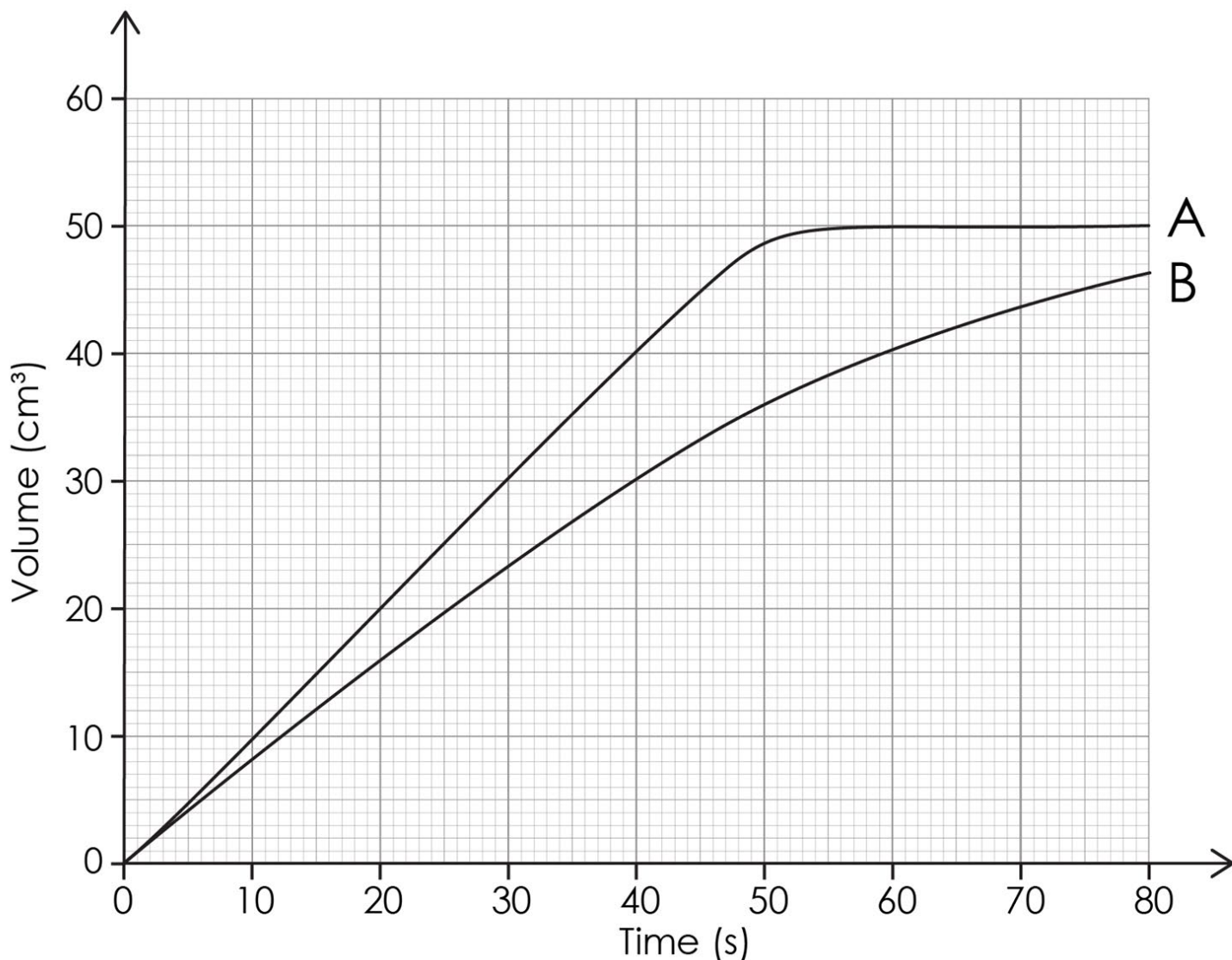


Calculate the mean rate of reaction between **0 and 40 seconds** for:

a) reaction A: $1 \text{ cm}^3/\text{s}$

b) reaction B

Calculating the gradient of a slope – answer b



Calculate the mean rate of reaction between **0 and 40 seconds** for:

b) reaction B

$$\text{difference in } y = 30 - 0 = 30$$

$$\text{difference in } x = 40 - 0 = 40$$

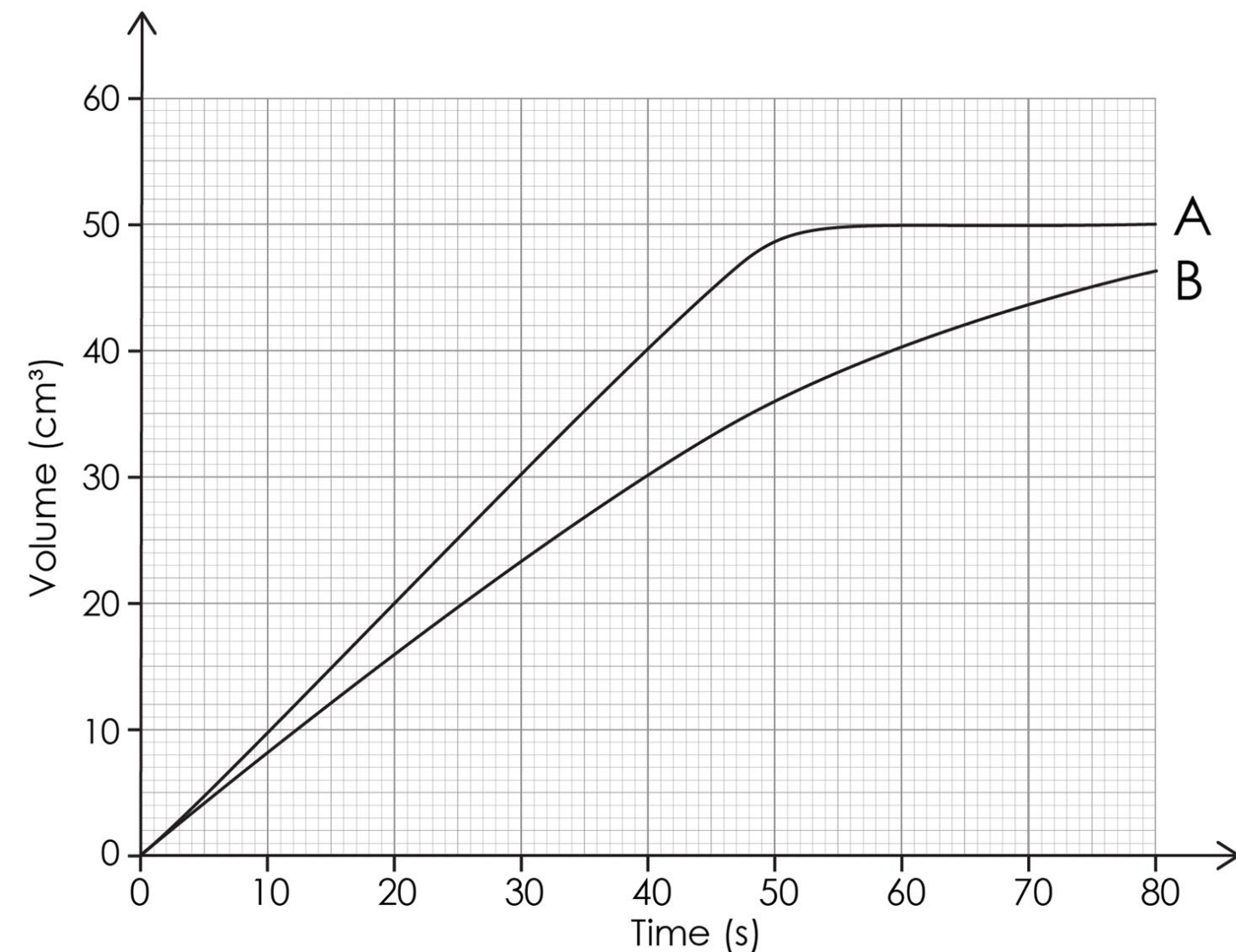
$$\text{gradient} = \frac{30}{40}$$

$$\text{gradient} = 0.75$$

$$\text{units} = \text{cm}^3/\text{s}$$

Answer: $0.75 \text{ cm}^3/\text{s}$

Comparing the mean rates of reaction



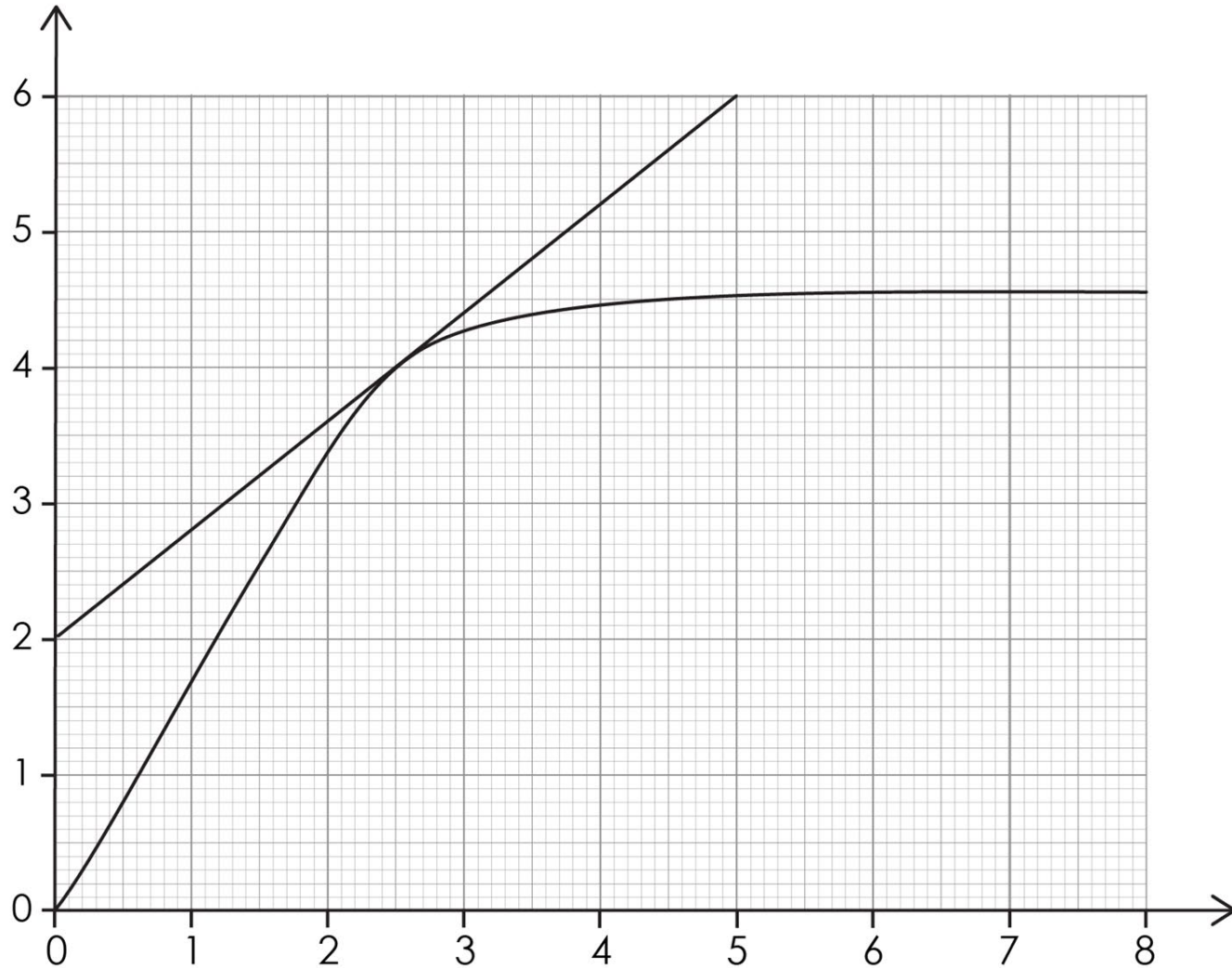
- a) Compare the mean rate of reaction between 0 and 40 seconds for reaction A and reaction B.
- b) Which line represents the reaction with larger lumps of sodium carbonate?
- c) Why do you think this is the case?

Comparing the mean rates of reaction – answers

- a) Reaction A has a faster rate of reaction between 0 and 40 seconds than reaction B. Reaction A has a rate of reaction of $1 \text{ cm}^3/\text{s}$ in the first 40 seconds, Reaction B has a reaction rate of $0.75 \text{ cm}^3/\text{s}$ in that same time. The steeper the slope, the faster the reaction.
- b) Line A represents the reaction with smaller lumps; line B represents the reaction with larger lumps.
- c) The smaller lumps will have a faster rate of reaction because they have a larger surface area, so the line is steeper.



Drawing a tangent to a curve

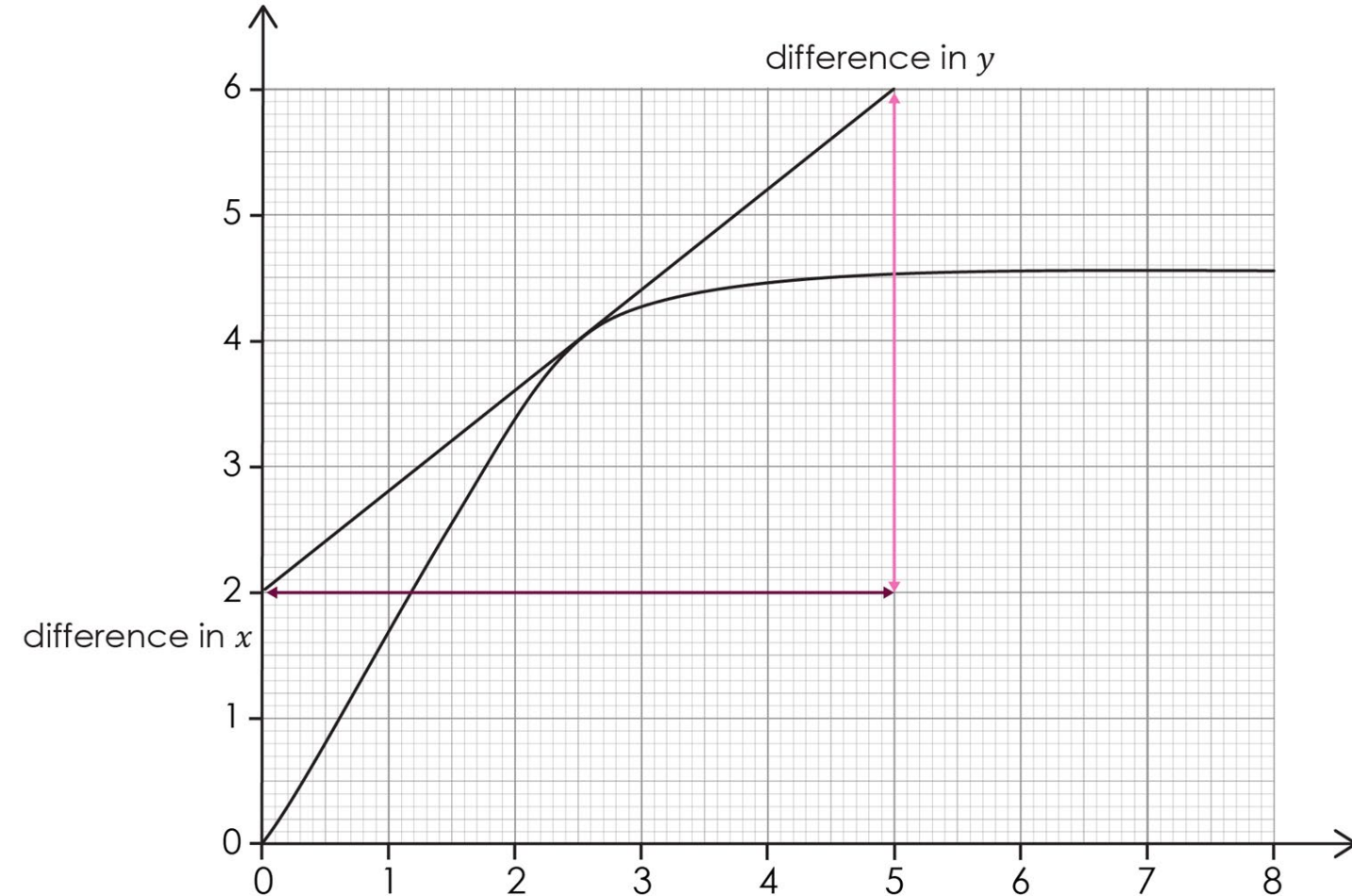


The gradient of a slope describes its steepness.

A tangent can be used to find the gradient of a curve at a specific point.

A tangent is a straight line that follows the general direction of the curve at that point.

Calculating the gradient of a tangent



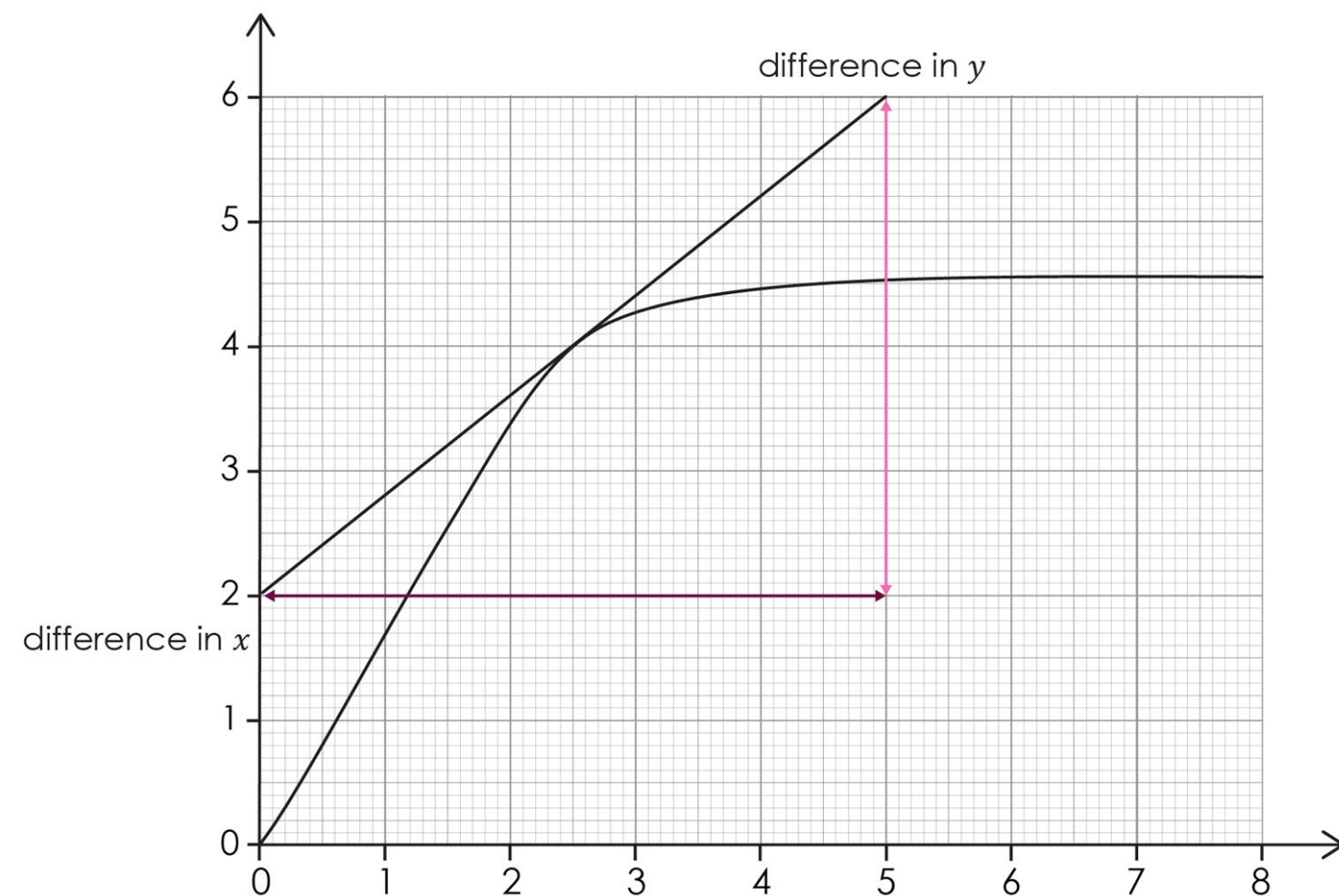
Step 1: Identify two points on the line.

Step 2: Find the difference in y and the difference in x .

Step 3: Substitute the values into the following equation:

$$\text{gradient} = \frac{\text{difference in } y}{\text{difference in } x}$$

Calculating the gradient of a tangent – example



Step 1: Identify two points on the line.

Step 2: Find the difference in y and the difference in x .

$$\text{difference in } y = 6 - 2 = 4$$

$$\text{difference in } x = 5 - 0 = 5$$

Step 3: Substitute the values into the following equation:

$$\text{gradient} = \frac{\text{difference in } y}{\text{difference in } x}$$

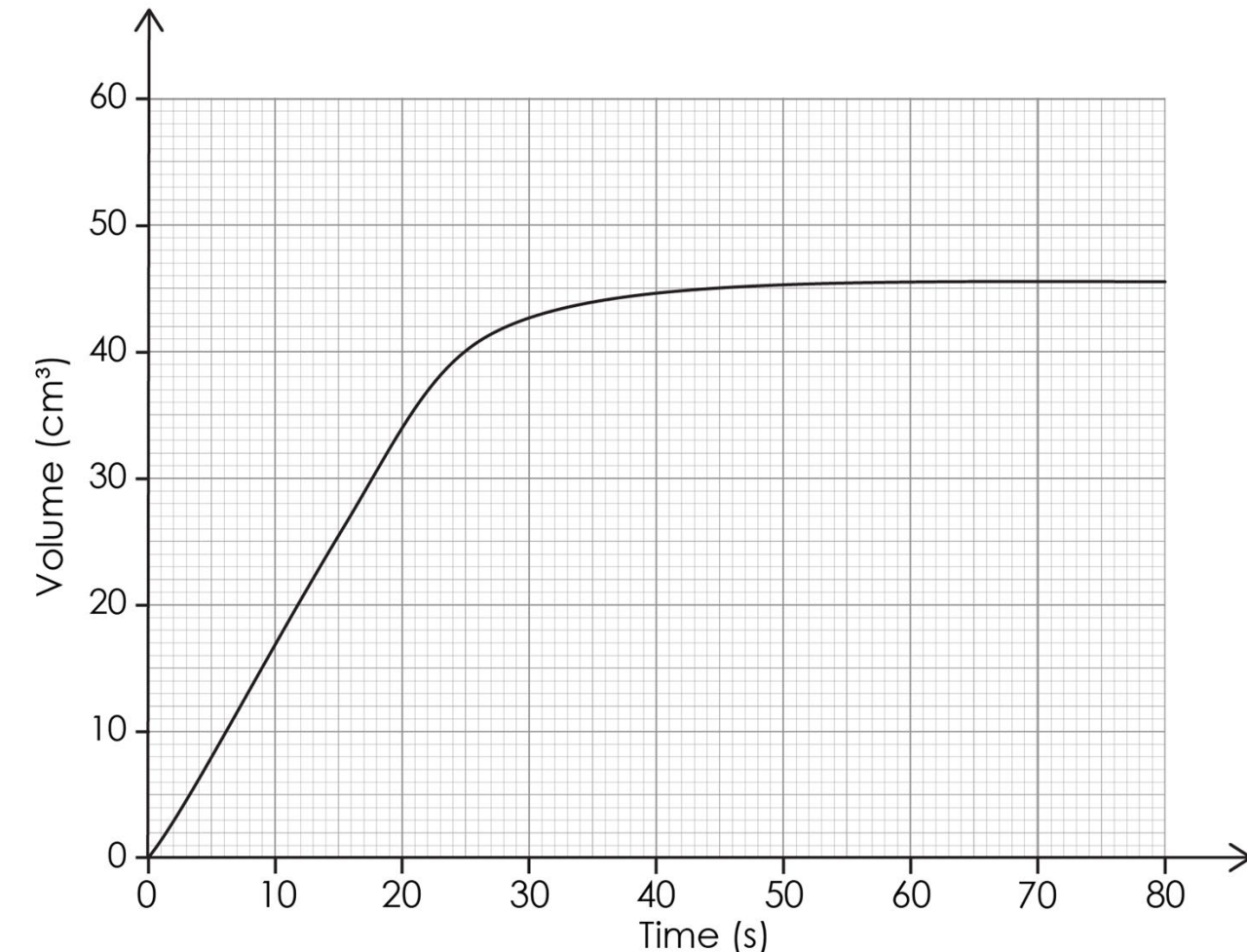
$$\text{gradient} = \frac{4}{5}$$

$$\text{gradient} = 0.8$$



Returning to lesson question 2

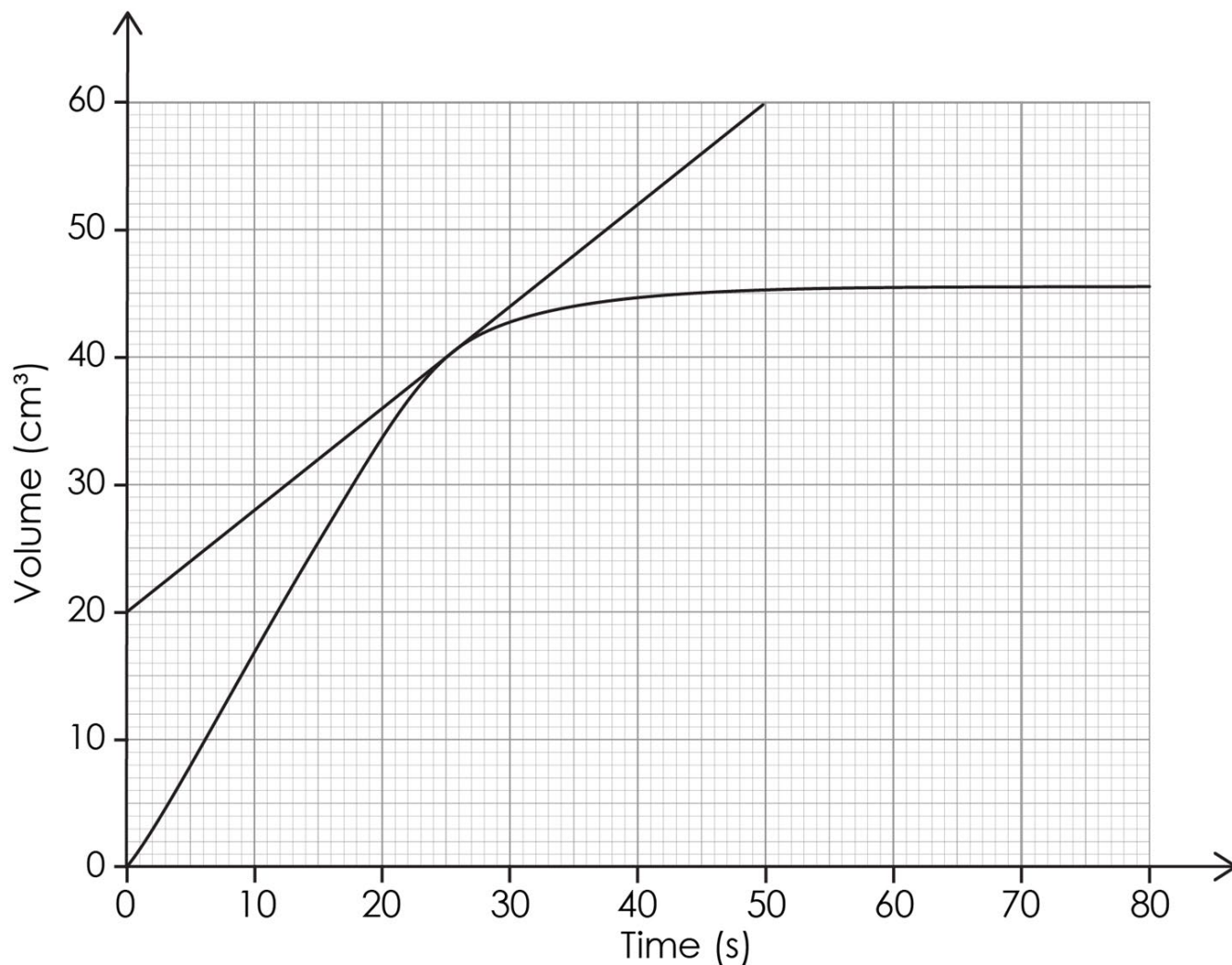
Let's go back to example we looked at on slide 24:



Imagine you are investigating the effect of changing the size of calcium carbonate lumps on the rate of the reaction with hydrochloric acid.

Determine the **rate of reaction** at **25 seconds**.

Calculating the gradient using a tangent



Draw a tangent to the line at 25 seconds.

Find the difference in y and the difference in x using the scales on the axis:

$$\text{difference in } y = 60 - 20 = 40 \text{ cm}^3$$

$$\text{difference in } x = 50 - 0 = 50 \text{ s}$$

Calculate the gradient:

$$\text{gradient} = \frac{\text{difference in } y}{\text{difference in } x}$$

$$\text{gradient} = \frac{40 \text{ cm}^3}{50 \text{ s}}$$

$$\text{gradient} = 0.8 \text{ cm}^3/\text{s}$$