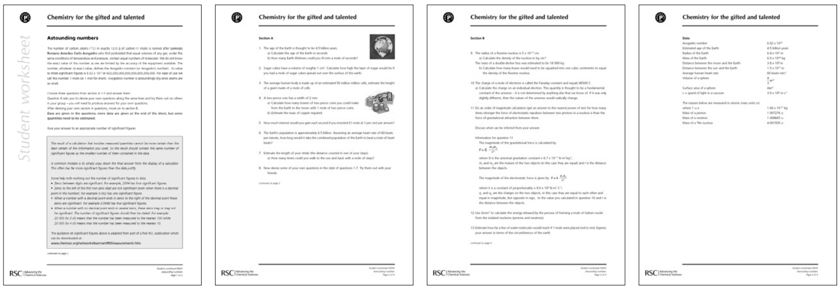
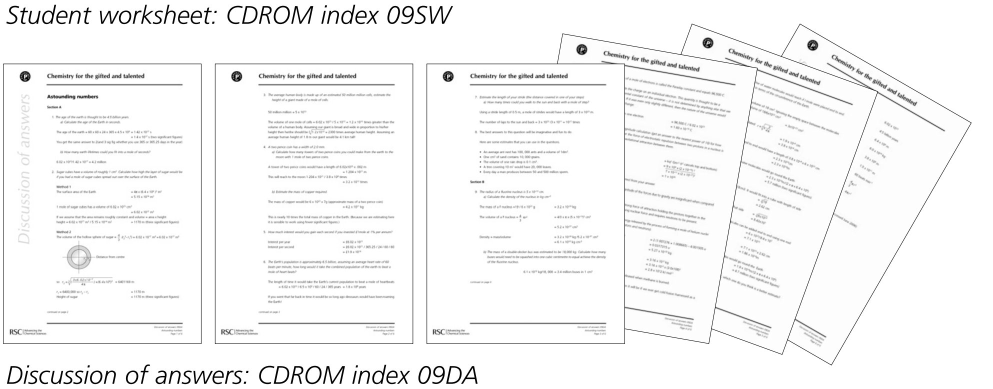
Astounding numbers



Student worksheet: CDROM index 09SW



Discussion of answers: CDROM index 09DA

Topics

Avogadro's number, the charge on an electron and proton, the density of a nucleus and the mass deficit of a helium nucleus.

Level

Very able pre-16 students or able post-16 students.

Prior knowledge

Students will need to be able to use standard form and rearrange equations.

Faraday's constant is quoted in Coulombs; students who have not met units of charge previously could leave out question 10.

Rationale

This activity is designed to be fun and generate a wow factor for the students. It tries to convey how amazing the scale is that we use in chemistry. In question 8 the students are asked to use creative thinking and come up with some questions of their own.

The students are asked to estimate some of their answers and they should appreciate that it may be desirable to work with fewer significant figures in these cases.

Use

This could be given to the more able students in a class, who have mastered mole calculations while the others are still practising.

More than one student needs to do the activity for them to try out their questions on each other. To finish off they could be asked to give a short presentation to the rest of the group along the lines of 'Did you know 1 mole of...'

There are some hints they can be given if they run out of ideas for question 8. Where a quantity needs to be estimated the students could be asked to try an internet search or simply guess – *eg* the number of grains of sand in 1 cm^3 could be estimated and the estimate written into the question in the same way that the mass of a double-decker bus is in question 9.

How big is the beach with a mole of sand grains?
How big is an omelette made with a mole of eggs?
How far could a snail crawl in a mole of seconds?
How big is an ocean with a mole of buckets?
How deep is the flood with a mole of raindrops?
How many men would it take to produce a mole of sperm?
How large is a forest with a mole of leaves?

The answer to question 12 in the *Discussion of answers* sheet

The answers from the two methods are different. Which one do you think is a better estimate? Why?

I slightly favour the second method (answer 4.7 million) because it does not assume that there is no empty space in the water. However, it does assume a regular structure and it may have been better to use the density of ice rather than of water to base the calculation on. The density of ice is approx 0.92 g cm^{-3} and gives a final answer, using method two, of 4.8 million.

Student worksheet

Astounding numbers

The number of carbon atoms (^{12}C) in exactly 12.0 g of carbon (1 mole) is named after **Lorenzo Romano Amedeo Carlo Avogadro** who first postulated that equal volumes of any gas, under the same conditions of temperature and pressure, contain equal numbers of molecules. We do not know the exact value of this number as we are limited by the accuracy of the equipment available. The number, whatever its exact value, defines the Avogadro constant (or Avogadro's number). Its value to three significant figures is 6.02×10^{23} or 602,000,000,000,000,000,000,000. For ease of use we call this number 1 mole (or 1 mol for short). Avogadro's number is astoundingly big since atoms are so small.

Choose three questions from section A 1-7 and answer them.
Question 8 asks you to devise your own questions along the same lines and try them out on others in your group – you will need to produce answers for your own questions.
After devising your own section A questions, move on to section B.
Data are given in the questions; more data are given at the end of the sheet, but some quantities need to be estimated.

Give your answer to an appropriate number of significant figures.

The result of a calculation that involves measured quantities cannot be more certain than the least certain of the information you used. So the result should contain the same number of significant figures as the smallest number of significant figures contained in the data.

A common mistake is to simply copy down the final answer from the display of a calculator. This often has far more significant figures than the data justify.

Some help with working out the number of significant figures in data:

- *Zeros between digits are significant. For example, 2004 has four significant figures.*
- *Zeros to the left of the first non-zero digit are not significant (even when there is a decimal point in the number). For example 0.002 has one significant figure.*
- *When a number with a decimal point ends in zeros to the right of the decimal point these zeros are significant. For example 2.0040 has five significant figures.*
- *When a number with no decimal point ends in several zeros, these zeros may or may not be significant. The number of significant figures should then be stated. For example: 20 000 (to 3 sf) means that the number has been measured to the nearest 100 while 20 000 (to 4 sf) means that the number has been measured to the nearest 10.*

The guidance on significant figures above is adapted from part of a free RSC publication which can be downloaded at
www.chemsoc.org/networks/learnnet/RSCmeasurements.htm.

continued on page 2

Section A

1. The age of the Earth is thought to be 4.5 billion years.
   a) Calculate the age of the Earth in seconds.
   b) How many Earth lifetimes could you fit into a mole of seconds?

2. Sugar cubes have a volume of roughly 1 cm^3. Calculate how high the layer of sugar would be if you had a mole of sugar cubes spread out over the surface of the earth.

3. The average human body is made up of an estimated 50 million million cells. Estimate the height of a giant made of a mole of cells.

4. A two pence coin has a width of 2 mm.
   a) Calculate how many towers of two pence coins you could make from the Earth to the moon with 1 mole of two pence coins.
   b) *Estimate* the mass of copper required.

5. How much interest would you gain each second if you invested a mole of £1s at 1 per cent per annum?

6. The Earth's population is approximately 6.5 billion. Assuming an average heart rate of 60 beats per minute, how long would it take the combined population of the Earth to beat a mole of heart beats?

7. Estimate the length of your stride (the distance covered in one of your steps). How many times could you walk to the sun and back with a mole of steps?

8. Now devise some of your own questions in the style of questions 1-7. Try them out with your friends.

continued on page 3

Section B

9. The radius of a fluorine nucleus is 5 x 10^{-13} cm.
   a) Calculate the density of the nucleus in kg cm^{-3}

   The mass of a double-decker bus was estimated to be 18 000 kg.
   b) Calculate how many buses would need to be squashed into one cubic centimetre to equal the density of the fluorine nucleus.

10. The charge of a mole of electrons is called the Faraday constant and equals 96 500 C. Calculate the charge on an individual electron. This quantity is thought to be a fundamental constant of the universe – it is not determined by anything else that we know of. If it was only slightly different, then the nature of the universe would radically change.

11. Do an order of magnitude calculation (get an answer to the nearest power of ten) for how many times stronger the force of electrostatic repulsion between two protons in a nucleus is than the force of gravitational attraction between them.

    Discuss what can be inferred from your answer.

    Information for question 11

    The magnitude of the gravitational force is calculated by:

    $$F = G\,\frac{m_1 m_2}{r^2}$$

    where *G* is the universal gravitation constant = 6.7 x 10^{-11} N m^2 kg^{-2},
    m_1 and m_2 are the masses of the two objects (in this case they are equal) and *r* is the distance between the objects.

    The magnitude of the electrostatic force is given by $F = k\,\frac{q_1 q_2}{r^2}$

    where *k* is a constant of proportionality = 9.0 x 10^9 N m^2 C^{-2},
    q_1 and q_2 are the charges on the two objects (in this case they are equal to each other and equal in magnitude, but opposite in sign, to the value you calculated in question 10 and *r* is the distance between the objects).

12. Use $E=mc^2$ to calculate the energy released by the process of forming one mole of helium nuclei from the isolated nucleons (protons and neutrons).

13. Estimate how far a line of water molecules would reach if one mole were placed end to end. Express your answer in terms of the circumference of the earth.

continued on page 4

Data

Avogadro's number	6.02×10^{23}
Estimated age of the Earth	4.5 billion years
Radius of the Earth	6.4×10^{6} m
Mass of the Earth	6.0×10^{24} kg
Distance between the moon and the Earth	3.8×10^{8} m
Distance between the sun and the Earth	1.5×10^{11} m
Average human heart rate	60 beats min^{-1}
Average human height	1.8 m
Mass of 2 pence coin	7 g
Volume of a sphere	$\frac{4}{3}\pi r^3$
Surface area of a sphere	$4\pi r^2$
c = speed of light in a vacuum	3.0×10^{8} m s^{-1}
The masses below are measured in atomic mass units (u) where 1 u =	1.66×10^{-27} kg
Mass of a proton	1.007276 u
Mass of a neutron	1.008665 u
Mass of a ^{4}He nucleus	4.001505 u

Astounding numbers

Section A

1. *The age of the Earth is thought to be 4.5 billion years.*
   a) Calculate the age of the Earth in seconds.

The age of the Earth = 60 x 60 x 24 x 365 x 4.5 x 10^9 = 1.42 x 10^{17} s
= **1.4 x 10^{17} s (two significant figures)**

You get the same answer to 2 (and 3) significant figures whether you use 365 or 365.25 days in the year.

b) How many Earth lifetimes could you fit into a mole of seconds?

6.02 x 10^{23}/1.42 x 10^{17} = **4.2 million**

2. *Sugar cubes have a volume of roughly 1 cm^3. Calculate how high the layer of sugar would be if you had a mole of sugar cubes spread out over the surface of the Earth.*

Method 1

The surface area of the Earth = 4π x (6.4 x 10^6)^2 m^2
= 5.15 x 10^{14} m^2

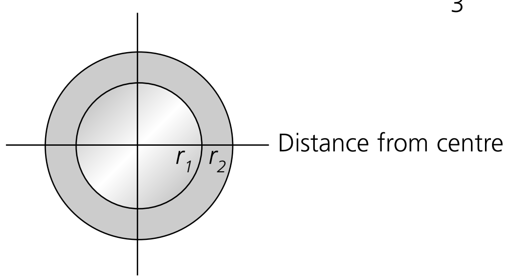
1 mole of sugar cubes has a volume of 6.02 x 10^{23} cm^3
= 6.02 x 10^{17} m^3

If we assume that the area remains roughly constant and volume = area x height

height = 6.02 x 10^{17} m^3 / 5.15 x 10^{14} m^2 = **1170 m (three significant figures)**

Method 2

The volume of the hollow sphere of sugar = $\frac{4}{3}\pi(r_2^3 - r_1^3)$ = 6.02 x 10^{17} m^3



so $r_2 = \sqrt[3]{\left(\frac{3 \times 6.02 \times 10^{17}}{4\pi}\right) + (6.4 \times 10^6)^3}$ = 6401170 m

r_1 = 6400,000 so $r_2 - r_1$ = 1170 m

Height of sugar = **1170 m (three significant figures)**

continued on page 2

3. *The average human body is made up of an estimated 50 million million cells. Estimate the height of a giant made of a mole of cells.*

50 million million = 5×10^{13}

The volume of one mole of cells = $6.02 \times 10^{23} / 5 \times 10^{13} = 1.2 \times 10^{10}$ times greater than the volume of a human body. Assuming our giant is broad and wide in proportion to his/her height then he/she should be $\sqrt[3]{1.2x10^{10}}$ = 2300 times average human height. Assuming an average human height of 1.8 m our giant would be **4.1 km tall**!

4. *A two pence coin has a width of 2.0 mm.*
   a) *Calculate how many towers of two pence coins you could make from the earth to the moon with 1 mole of two pence coins.*

A tower of two pence coins would have a length of $6.02 \times 10^{23} \times 0.002$ m
= 1.204×10^{21} m
This will reach to the moon $1.204 \times 10^{21} / 3.8 \times 10^{8}$ times
= **3.2×10^{12} times**

   b) *Estimate the mass of copper required.*

The mass of copper would be $6 \times 10^{23} \times 7$g (approximate mass of a two pence coin)
= **4.2×10^{21} kg**

This is nearly 10 times the total mass of copper in the Earth. (Because we are estimating here it is sensible to work using fewer significant figures.)

5. *How much interest would you gain each second if you invested a mole of £1s at 1% per annum?*

Interest per year	= £6.02×10^{21}
Interest per second	= £6.02×10^{21} / 365.25 / 24 / 60 / 60
	= **£1.9×10^{14}**

6. *The Earth's population is approximately 6.5 billion. Assuming an average heart rate of 60 beats per minute, how long would it take the combined population of the earth to beat a mole of heart beats?*

The length of time it would take the Earth's current population to beat a mole of heartbeats
= $6.02 \times 10^{23} / 6.5 \times 10^{9}$ / 60 / 24 / 365 years = **1.8×10^{8} years**

If you went that far back in time it would be so long ago dinosaurs would have been roaming the Earth!

continued on page 3

7. *Estimate the length of your stride (the distance covered in one of your steps). How many times could you walk to the sun and back with a mole of steps?*

   Using a stride length of 0.5 m, a mole of strides would have a length of 3×10^{23} m.

   The number of laps to the sun and back = $3 \times 10^{23} / 3 \times 10^{11}$ = **10^{12} times**

8. The best answers to this question will be imaginative and fun to do.

   Here are some *estimates* that you can use in the questions.

   - An average ant nest has 100 000 ants and a volume of 1 dm^3.
   - One cm^3 of sand contains 10 000 grains.
   - The volume of one rain drop is 0.1 cm^3.
   - A tree covering 10 m^2 would have 20 000 leaves.
   - Every day a man produces between 50 and 500 million sperm.

Section B

9. *The radius of a fluorine nucleus is 5×10^{-13} cm.*
   a) Calculate the density of the nucleus in kg cm^{-3}

   The mass of a F nucleus =19 / 6 x 10^{23} g = 3.2×10^{-26} kg

   The volume of a F nucleus = $\frac{4}{3}\pi r^3$ = $4/3 \times \pi \times (5 \times 10^{-13})^3$ cm^3

   = 5.2×10^{-37} cm^3

   Density = mass/volume = 3.2×10^{-26} kg $/5.2 \times 10^{-37}$ cm^3
   = **6.1×10^{10} kg cm^{-3}**

   b) The mass of a double-decker bus was estimated to be 18 000 kg. Calculate how many buses would need to be squashed into one cubic centimetre to equal achieve the density of the fluorine nucleus.

   6.1×10^{10} kg/18 000 = **3.4 million buses in 1 cm^3**

continued on page 4

10. *The charge of a mole of electrons is called the Faraday constant and equals 96 500 C. Calculate the charge on an individual electron. This quantity is thought to be a fundamental constant of the universe – it is not determined by anything else that we know of. If it was even only slightly different, then the nature of the universe would radically change*.

The charge on one electron	= 96,500 C / 6.02 x 10^{23}
	= **1.60 x 10^{-19} C**

11. *Do an order of magnitude calculation (get an answer to the nearest power of 10) for how many times stronger the force of electrostatic repulsion between two protons in a nucleus is than the force of gravitational attraction between them.*

The ratio of forces	= kq^2 Gm^{-2} (r^2 cancels top and bottom)
	= $\frac{9 \times 10^{9} \times (2 \times 10^{-19})^2}{7 \times 10^{-11} \times (2 \times 10^{-27})^2}$
	= **1 x 10^{36}**

Discuss what can be inferred from your answer.

At an atomic scale the magnitude of the forces due to gravity are insignificant when compared to the electrostatic forces.

There must be some other strong force of attraction holding the protons together in the nucleus. This is called the strong nuclear force and requires neutrons to be present.

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12. *Use $E= mc^2$ to calculate the energy released by the process of forming a mole of helium nuclei from the isolated nucleons (protons and neutrons).*

The mass deficit for one nucleus	= 2 (1.007276 + 1.008665) – 4.001505 *u*
	= 0.0317315 *u*
	= 5.27 x 10^{-29} kg
The mass deficit for one mole	= 3.16 x 10^{-5} kg
Energy released	= 3.16 x 10^{-5} x $(3.0 \times 10^{8})^2$
	= **2.8 x 10^{12} kJ mol^{-1}**

Compare this with the 890 kJ mol^{-1} released when methane is burned.

What a leap forward in energy provision it will be if we ever get cold fusion harnessed as a power source!

continued on page 5

13. *Estimate how far a line of water molecules would reach if one mole were placed end to end. Express your answer in terms of the circumference of the Earth.*

Method 1

One mole of water has a volume of 18 cm^3. Ignoring the empty space between the molecules, each one should have a volume of $18/6 \times 10^{23}$ cm^3

$= 3 \times 10^{-23}$ cm^3

The radius can then be calculated $r = \sqrt[3]{V\frac{3}{4\pi}}$ $= 1.9 \times 10^{-8}$ cm

Therefore the diameter is $= 3.8 \times 10^{-8}$ cm

One mole of these molecules end to end would have a length of $3.8 \times 10^{-8} \times 6 \times 10^{23}$ cm

$= 2.3 \times 10^{16}$ cm

$= 2.3 \times 10^{14}$ m

The number of times the line of water molecules would go round the Earth

$= 2.3 \times 10^{14}$ m$/(2 \times \pi \times 6.4 \times 10^6)$

= **5.7 million (two significant figures)**

Method 2

One mole of water has a volume of 18cm^3. It would fit into a cube with length of side

$= \sqrt[3]{18}$

$= 2.62$ cm

The number of water molecules along that side $= \sqrt[3]{6 \times 10^{23}}$

$= 8.43 \times 10^7$

The number of times that number of molecules can be added end to end using one mole

$= 6 \times 10^{23}/8.4 \times 10^7$

$= 7.1 \times 10^{15}$

The distance covered by those molecules $= 7.1 \times 10^{15} \times 2.62$ cm

$= 1.86 \times 10^{14}$ m

The number of times the line of water molecules would go round the Earth

$= 1.9 \times 10^{14}$ m$/(2 \times \pi \times 6.4 \times 10^6)$

= **4.7 million (two significant figures)**

The answers from the two methods are different, which one do you think is a better estimate? Why?

continued on page 6

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Mass of a ^{4}He nucleus	4.001505 u

For more information about Avogadro visit:
http://www.bulldog.u-net.com/avogadro/avoga.html (accessed June 2006).