Argon

Introduction
Teachers who have not used the problems before should read the section Using the problems before starting.

Prior knowledge
Familiarity with Avogadro's number, the characteristics of solids, liquids and gases and the concept of atomic size. A detailed knowledge is unnecessary as students are encouraged to consult textbooks and data books during the exercise.

Resources
Scientific calculators, suitable data books should be available for reference.

Group size
2–3.

Possible solutions
The key question which underpins this problem is 'What is meant by the volume of a gas?' Research has shown that students have difficulty imagining an empty space or a vacuum, and perceive matter as being continuous with no empty space. The problem can be approached on at least two levels.

a. By calculating the space between the atoms. This can be done starting from atomic sizes or from the density of the substance as a solid or as a liquid, and assuming that the atoms touch one another.
b. By calculating the space within the atoms and between the atoms. This can be done starting from the sizes of the nucleus and of the electrons. The data are not easy to find and it is probably best not to introduce this method unless students think of it. It could be mentioned at the very end of the problem. The calculation is set out below.

Method a. Calculating the space between atoms
There are at least two alternative approaches:

(i) From atomic sizes
The van der Waals radius of argon is 0.192 nm (Stark and Wallace 1989)

Volume of this sphere = $4/3\ \pi r^3 = 4/3\ \pi\ (0.192 \times 10^{-9})3\ m^3$

$= 2.96 \times 10^{-29}\ m^3$

volume of 1 mole of argon atoms = $6.02 \times 1023 \times 2.96 \times 10^{-29}\ m^3$

$= 1.78 \times 10^{-5}\ m^3$

= 18 cm^3 (to two significant figures)
This figure is for the atoms alone and does not include the spaces between them.

(ii) From the density of the liquid
The density of argon liquid at –186 °C (its boiling point) is 1.40 g cm^{-3} (Stark and Wallace); and 1 mole of argon atoms has a mass of 39.948 g (40 g to two significant figures).
Therefore 1 mole of argon atoms occupies 40/1.4 = 29 cm^3 ie 6.0×10^{23} atoms of argon occupy 29 cm^3 when touching one another.

The 18 cm^3 figure is a minimum estimate as the spaces between the spherical argon atoms in the liquid have been ignored. The atoms are close packed: reference to a textbook gives the information that 74 % of the available space in a close packed array of spheres is occupied. It seems an unnecessary complication to introduce this figure, but 74 % of 29 cm^3 is 21 cm^3 – close to the 18 cm^3 calculated above.

The subsequent calculation is the same for both methods using whatever value the students found. At 0 °C and normal atmospheric pressure, 1 mole of gaseous argon atoms occupies 22 400 cm^3
The percentage of 'filled' space in argon = 18/22 400)x 100 %
(or 29/22 400 x 100 %)
= 0.08 % (0.13 %)
and the percentage of 'empty' space = 99.92 % (99.87 %).

Suggested approach

During trialling the following instructions were given to students and proved to be extremely effective:

1. Working as a group, discuss the problem and try to work out the answer. You can divide the work amongst yourselves if you wish but keep one another informed of progress.
   Discussion can play a vital part in working out solutions to problems like this. Several minds working together on a problem can stimulate ideas that each one on its own could not manage. About 10 minutes should be spent on the initial discussion with further discussion as required.
2. Write a brief account of what you did.
3. Working as a group, prepare a short (ca 5-minute maximum) presentation suited to middle secondary pupils, but to be delivered to the rest of your class. If possible all group members should take part: any method of presentation (such as a blackboard, overhead projector, etc) can be used. Outline the problem at the start of the presentation and then describe what you did. After the presentation, be prepared to accept and answer questions from the rest of the class and discuss with them what you did.

Possible extensions

1. If argon atoms could be made into a single line, how far would one mole of them stretch?
As in **a.(i)**, for argon atoms, r = 0.192 nm, and d = 0.384 nm
= 0.384 x 10^{-9} m, or 4 x 10^{-10} m

One mole of these will stretch 6 x 1023 x 4 x 10^{-10} m
= 2.4 x 10^{14} m (2.4 x 10^{11} km)

Compare this figure with:

- the diameter of the earth = 1.3 x 10^7 m;
- the diameter of the sun = 1.4 x 10^9 m;
- earth – sun distance = 1.5 x 10^{11} m; and
- the distance to the nearest star (Proxima Centauri) = 1.5 x 10^{17} m.

This large distance demonstrates just how long a single line of atoms would be.

2. Compare argon with other elements – for example with helium and xenon.
The following figures are calculated on the basis of the answer in **a.(ii)**.

Helium

density = 0.14 g cm^{-3}; mass of 1 mole = 4.00 g
volume of 1 mole = 4.00/0.147 = 27.2 cm^3
as above, percentage of 'filled' space in helium = 27.2/22 400 x 100 %
= 0.12 %
and the percentage of 'empty' space = 99.88 %

Xenon

density = 3.52 g cm^{-3}; mass of 1 mole = 131 g
volume of 1 mole = 131/3.52 = 37.2 cm^3
as above, percentage of 'filled' space in xenon = 37.2/22 400 x 100 %
= 0.17 %
and the percentage of 'empty' space = 99.83 %

ie the 'empty space' in helium, argon and xenon is 99.88 %, 99.87 %, and 99.83 % respectively. The closeness of these figures demonstrates that the volume of the molecules in any gas is a very small proportion of the whole.

How to put this across?

An example is needed, for example:

- in a litre of argon at 0°C and normal atmospheric pressure only 1 cm^3 is made up of atoms; and
- in a school of 1000 pupils, only 1 would be present.

Background information

Method b. Calculating the space within and between atoms

The figures here are not easily accessible. The nuclear radius is given by:
$r= r_0A^{1/3}$

(where r_0 is a constant with value ca 1.3 x 10^{-13} cm) and A is the mass number of the atom

Argon has mass number 40, so r = 4.4 x 10^{-13} cm

Volume of this sphere = 4/3 πr^3 = 4/3 π (4.4 x 10^{-13})^3 cm^3
= 8.5 x 10^{-38} cm^3

Volume of 1 mole of argon nuclei = 6.0 x 10^{23} x 8.5 x 10^{-38} cm^3
= 5.0 x 10^{-14} cm^3

Carrying this figure forward as above gives the percentage of 'filled' space in argon as
5.0 x 10^{-14}/22 400 x 100 %
= 2 x 10^{-16} %

The percentage of 'empty' space is just about as near 100% as you can get.
Adding in electron sizes would increase this figure slightly. However, not even the nucleus is 'solid'! The same textbook states 'although nuclear densities are high compared with ordinary matter, nuclei are by no means close packed with nucleons … Individual nuclei can travel rather freely through nuclear matter'.

Notes

1. It is difficult to comprehend the sheer size of Avogadro's number. Peter Fensham, in an article in Chemeda, The Australian Journal of Chemical Education (1991, July, 3),

tried to put this into perspective. He quotes the following example taken from J. Chem. Ed., 1990, **67**, 481. 'A mole of pound coins was created when the earth was formed 4 500 million years ago. The coins have been given away at the rate of one million per second since that time. When did the money run out? If it has not run out, how much remains? Calculation shows that the money has not yet run out and that 0.76 mol of coins remain.'

2. It is essential that the following data for argon are available – van der Waals radii, density of the liquid or solid, density of the gas, molar volume of an ideal gas (22.4 dm^{-3}). In Scotland, for example, the revised H-grade data books do not have this information but the old ones do. Data books with too much information have also been unsatisfactory.
3. See for example B. Anderson, Studies in Science Education, 1990, **17**, 53.
4. J.G. Stark, H.G. Wallace, A chemistry data book, John Murray, London, 1989.
5. The ideal gas equation PV = nRT assumes no attraction between gas molecules and zero volume of these molecules. The van der Waals equation takes these two factors into account. For one mole of gas:

$(P + a/V^2)\ (V - b) = RT$

where a is a measure of the attractive force between molecules, and is significant at low pressures (units: $dm^3\ atm\ mol^{-2}$); and b is related to the volume of the molecules themselves, and is significant at high pressures (units $dm^3\ mol^{-1}$).

Gas	He	Ar	Ne	Xe	O_2	CO_2	SO_2	CH_4	n-butane
a	0.034	1.34	0.21	4.2	1.36	3.6	6.7	2.3	14.5
b (x 100)	2.4	3.2	1.7	5.1	3.2	4.3	5.6	4.3	12

Students who have met this equation might be interested to see how closely the two equations match at ordinary temperatures and pressures. Consider 1 mole of argon at 0°C and 1 atmosphere pressure:

a. Using $PV = RT$

$V = RT/P = 0.0821 \times 273/1 = 22.41\ dm^3$

b. Using $(P + a/V^2)\ (V - b) = RT$, and assuming 22.41 dm^3 for V in the denominator in term 1:

$(P + 1.34)/(V - 0.032) = 0.0821 \times 273$

$V = 22.32\ dm^3$

Reiterating the calculation with this value of V in the denominator in term 1 gives the same answer.

The accepted figure for the molar volume of an ideal gas is 22.4136 $dm^3\ mol^{-1}$.

6. Students will have to be given much of the information that follows.
7. Friedlander et al,Nuclear and radiochemistry, 3rd ed, chap 2. London: John Wiley, 1981.

Argon

Estimate the percentage of 'empty space' in a sample of gaseous argon.

This problem gives little information, and at first it may appear impossible to solve, but it can be solved. Scientific calculators and suitable data books are required.

You should refer to any sources of information that you think might help such as your notebooks, textbooks and data books. Ask for assistance if you get stuck.